The Scattering of Light from Material Surfaces

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1 Introduction

Human beings can see, in part, because certain neurons in our eyes change their membrane voltage in response to being hit by photons. The optics (corneas, pupils, and lenses) of our eyes serve to focus the incoming light in a perspective projection so that light reflected or emitted from a spot on a material surface is collected in a single spot on the retina. Most of the light that impinges on the retina has been scattered by opaque surfaces. Thus, to understand image formation, one should understand the scattering of light from material surfaces.

The scattering problem is a boundary value problem of Maxwell's equations. In general this problem is extremely complex. But certain simplifications can be made that enable us to find an expression for an approximate solution.

2 Geometry

Let a 3D Euclidean space be indexed by coordinates x', y', and z'. Let $\zeta = \zeta(x', y')$ define a differentiable surface over a compact region R of the $x'y'$-plane. Then, at each point (x', y') in the interior of R, surface ζ has a unit normal, $\hat{n} = \hat{n}(x', y')$. Assume there exists a point source of light in direction $\hat{s}$ at a distance, much greater than $\max_R\{|\zeta(x', y')|\}$. (See figure 1.) Then, light is incident on the surface with parallel, planar wave-fronts in direction $\hat{k}_1 = -\hat{s}$. Assume the surface is viewed from direction $\hat{k}_2$. The source direction and the view direction are constant with respect to (x', y'), but the surface normal direction varies as a function of (x', y'). Define

$$\vec{v} = \hat{k}_1 - \hat{k}_2. \tag{1}$$

Vector $-\vec{v}$ is midway between $\hat{s}$ and $\hat{k}_2$. If at a specific point, $(x', y', \zeta(x', y'))$, in space on the surface the surface normal, $\hat{n}(x', y')$, is in the opposite direction from $\vec{v}$, then the view direction is the specular direction, and we say that the surface is in the specular orientation at that point.

The scattering model that we will use estimates the radiant intensity (in direction $\hat{k}_2$) of the surface at a point as a function of $\vec{v}$ decomposed into coordinates with respect to the

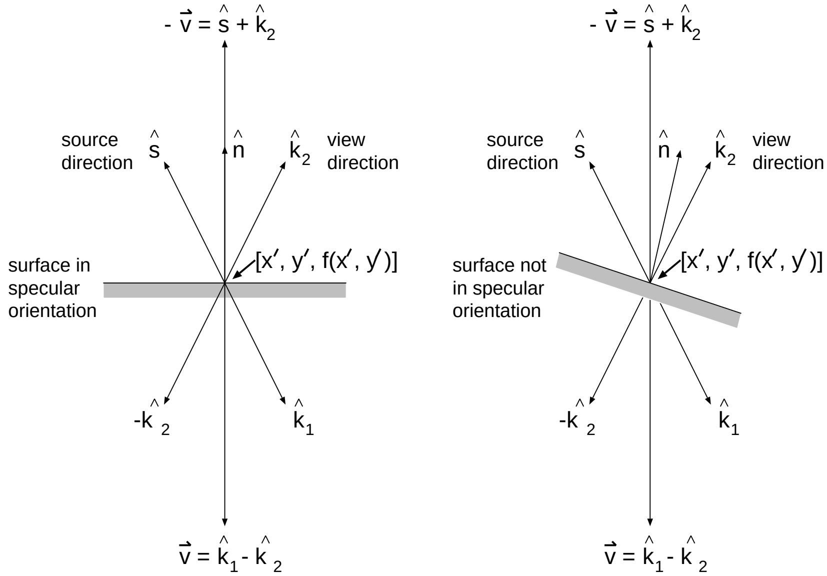

Figure 1: Vector $\vec{v}$ and the source and view directions. When the surface normal, $\hat{n}$, is in the same direction as $-\vec{v}$, the view direction is specular.

surface normal, $\hat{n}$. At each point of the surface the z-coordinate axis, $\hat{u}_z$, is defined by the outward normal direction, $\hat{n}$ (*i.e.*, $\hat{u}_z \equiv \hat{n}$). The x-axis, $\hat{u}_x$, is defined to point away from the light source so that the xz-plane contains both the surface normal and the incident light vector. That is, the xz-plane contains vectors $\hat{n}$, $\hat{s}$, $\hat{k}_1$, $\hat{u}_z$, and $\hat{u}_x$. Note that $\hat{u}_z = \hat{n}$ (they are the same), $\hat{u}_z \cdot \hat{u}_x = 0$ (they are perpendicular) and that $\hat{k}_1 = -\hat{s}$ (they are opposite). The y-axis, $\hat{u}_y$ is normal to that plane in the right-hand rule direction from $\hat{s}$ to $\hat{n}$.

At each surface point, vector $-\vec{v}$ is specified as a set of three coordinates with respect to this local coordinate system. Thus, if the surface is oriented in the specular direction, then $\vec{v} = v_z \hat{u}_z$ and $v_x = v_y = 0$.

To review, unit vectors $\{\hat{u}_x, \hat{u}_y, \hat{u}_z\}$ are defined in terms of the surface normal, $\hat{n}$, and the direction of incidence. $\hat{k}_1$, by

$$\hat{u}_z = \hat{n}, \tag{2}$$

$$\hat{u}_x = \hat{k}_1 - \left(\hat{n} \cdot \hat{k}_1\right) \hat{n} \;\Big/\; \left|\hat{k}_1 - \left(\hat{n} \cdot \hat{k}_1\right) \hat{n}\right|, \tag{3}$$

$$\hat{u}_y = \hat{n} \times \hat{u}_x. \tag{4}$$

Figure 2 depicts the geometry for an example surface point.

We will also need to refer to the angles of incidence and reflection. Define θ_i to be the angle of incidence, that is, the angle from the surface normal, $\hat{n}$ to the light source direction vector, $\hat{s}$. Let θ_r be the polar view-angle (or polar angle of reflection), the angle from $\hat{n}$ to the view direction vector, $\hat{k}_2$. Let ϕ_r be the azimuthal view angle, the angle from the xz-plane (which includes $\hat{k}_1$ and $\hat{n}$) to the plane of $\hat{n}$ and $\hat{k}_2$. These are shown in figure 3. In terms of the surface normal, the light source direction, the view direction, and the x and

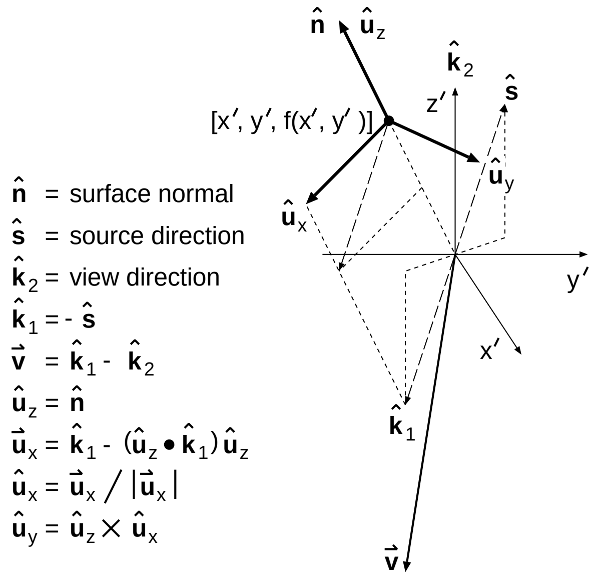

Figure 2: Global and local coordinate systems for scattering equations.

y unit vectors, the angles are

$$\cos\theta_i = \hat{s} \cdot \hat{n}, \tag{5}$$

$$\cos\theta_r = \hat{k}_2 \cdot \hat{n}, \tag{6}$$

$$\cos\phi_r = \left(\vec{k}_{2xy}/|\vec{k}_{2xy}|\right) \cdot \hat{u}_x, \tag{7}$$

where

$$\vec{k}_{2xy} = (\hat{k}_2 \cdot \hat{u}_x)\,\hat{u}_x + (\hat{k}_2 \cdot \hat{u}_y)\,\hat{u}_y. \tag{8}$$

3 The Helmholtz Solution for the Scattering of Light

Represent the electric field of an incident light wave by $\vec{E}_1$ and the electric field of the scattered wave by $\vec{E}_2$. Assume $\vec{E}_1$ is a harmonic plane wave of wavelength, λ, and of unit amplitude:

$$E_1 = e^{i(\vec{k}_1 \cdot \vec{r}' - \omega t)}. \tag{9}$$

$\vec{k}_1$ is the propagation vector,

$$\vec{k}_1 = \frac{2\pi}{\lambda}\hat{k}_1, \tag{10}$$

which is assumed to lie in the xz-plane. $\vec{r}'$ is the vector coordinate of the point in space at which the field is measured. For points on surface ζ,

$$\vec{r}' = x'\hat{u}_{x'} + y'\hat{u}_{y'} + \zeta(x', y')\hat{u}_{z'}. \tag{11}$$

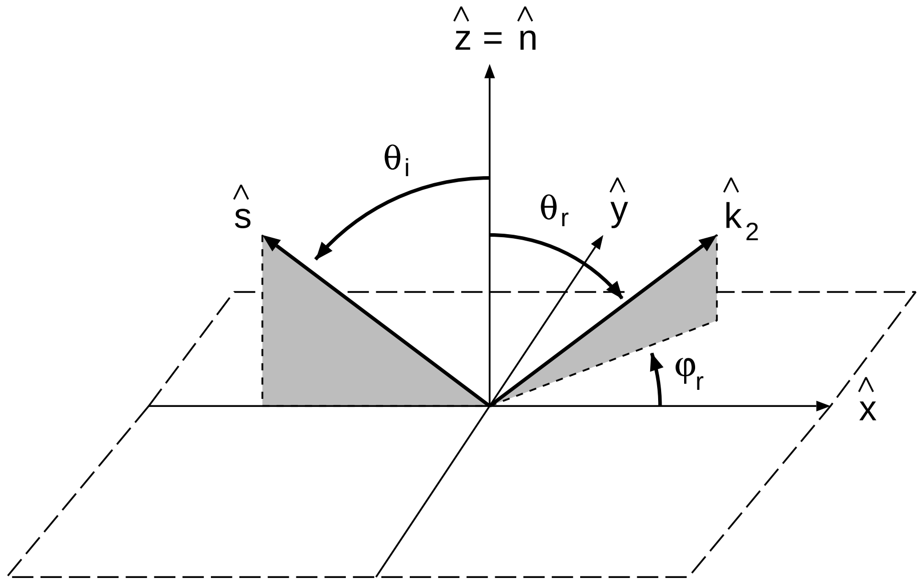

Figure 3: Angles of incidence and reflection in terms of vectors $\hat{s}$, $\hat{n}$, and $\hat{k}_2$.

All energy is assumed to be scattered at the original wavelength, therefore the propagation vector for the scattered wave is

$$\vec{k}_2 = \frac{2\pi}{\lambda}\hat{k}_2. \tag{12}$$

Note that if we define k in terms of $\vec{k}_j$ where $j = 1, 2$,

$$k \equiv k_1 = |\vec{k}_1| = k_2 = |\vec{k}_2| = \frac{2\pi}{\lambda}, \tag{13}$$

then k is the scalar propagation constant also known as β:

$$k = k_1 = k_2 = \beta = \omega\sqrt{\epsilon\mu}. \tag{14}$$

Let $\vec{P}$ be an observation point and let $\vec{R}'$ be the vector from $\vec{P}$ to the point $\vec{r}$, on surface S. Then the intensity, E_2, of the scattered field at $\vec{P}$ is given by the Helmholtz integral:

$$E_2(\vec{P}) = \frac{1}{4\pi}\int\int_S \left(E\frac{\partial\psi}{\partial n} - \psi\frac{\partial E}{\partial n}\right) \mathrm{d}S, \tag{15}$$

where

$$\psi = \frac{e^{ik_2R'}}{R'}. \tag{16}$$

The solution to this equation as a function of S is hard under most circumstances. Therefore certain simplifications are made to find approximate solutions. For example, we will consider a rough surface to have a statistical model.

Since the surface has a distribution of heights above the $x'y'$-plane with certain probabilities of occurrence, the surface height can be described by a random variable. This, in turn, causes the scattered electric field to be a random variable. It is easier to solve equation (15) for the expected value of the field, than for an exact value. In fact, what we solve for is the expected value of the squared magnitude of the field denoted $\langle E_2 E_2^* \rangle = \langle |E_2|^2 \rangle$. The magnitude is function of the angle of incidence, θ_i, and the angles, θ_r and ϕ_r, of reflection. The expected value can be written in terms of a reflection coefficient (or scattering coefficient), $\rho(\theta_i, \theta_r, \phi_r)$, by

$$\langle E_2 E_2^* \rangle = \langle |E_{20}|^2 \rangle \langle |\rho|^2 \rangle, \tag{17}$$

where E_{20} is the electric field reflected in the specular direction, $\theta_r = \theta_i$, by a smooth, perfectly conducting plane of the same dimensions, under the same angle of incidence, at the same distance, when the incident wave is horizontally polarized. E_{20} is deterministic, so all of the randomness is within the scattering coefficient. Thus, we can write

$$\langle E_2 E_2^* \rangle = |E_{20}|^2 \langle |\rho|^2 \rangle. \tag{18}$$

4 Surface Radiance

The electric field is, of course, only part of the picture. We do not directly perceive an electric field. We see that an area of surface has a particular brightness (as well as other attributes). That perceived brightness is related to the electric field through a radiance equation. The radiance of an infinitesimal surface patch of area dA is the flux emitted per unit foreshortened area per unit solid angle [2],

$$dL_r = \frac{d^2\Phi_r}{dA \cos\theta_r \; d\omega_r}, \tag{19}$$

where $d^2\Phi_r$ is the flux radiated into an infinitesimal solid angle, $d\omega_r$, at view angle, θ_r, and $dA\cos\theta_r$ is the apparent infinitesimal area as viewed from θ_r. (Here we are assuming that $\phi_r = 0$.) If the illuminated surface is planar with area A, the apparent area of the surface as viewed from angle θ_r is $A\cos\theta_r$, and if the distance, R_0, from the surface to the viewpoint is much larger than $\sqrt{A}$ then θ_r is approximately constant over the surface. Thus the luminant flux (due to the entire surface) into the infinitesimal solid angle at θ_r is

$$L_r = \frac{Ad^2\Phi_r}{d\omega_r}. \tag{20}$$

(Integrate $dL_r \, dA$ over A.) If for every view angle, θ_r, the light sensor has apparent area, A_s, as viewed from the surface, then the solid angle subtended by the sensor is $d\omega_r = A_s/R_0^2$. Thus the total irradiance of the light sensor is

$$L_r = \frac{AR_0^2 \, d^2\Phi_r}{A_s}. \tag{21}$$

The flux, $d^2\Phi_r$, is the light energy received by the sensor; it is dependent on the amplitude, E_{10}, of the electric field incident on the surface at point $\vec{P}$,

$$\vec{E}_1(\vec{P}) = E_{10}\, e^{[i\,(\vec{k}_1\cdot\vec{P})-\omega t]}\hat{k}_1, \tag{22}$$

and on the scattering coefficient, ρ.

$$d^2\Phi_r = \frac{1}{2}\sqrt{\frac{\mu}{\epsilon}}\mid E_{20}\mid^2 \langle\mid\rho\mid^2\rangle. \tag{23}$$

μ and ϵ are the permeability and permitivity of the medium. $\sqrt{\mu/\epsilon}$ is the intrinsic impedance of the medium. Recall that E_{20} is the field reflected in the specular direction by a smooth, perfectly conducting plane of the same dimensions under the same angle of incidence at the same distance, when the incident wave is horizontally polarized. Thus,

$$E_{20} = E_{10}\frac{ike^{ikR_0}A\cos\theta_i}{\pi R_0} = E_{10}\frac{2ie^{ikR_0}A\cos\theta_i}{\lambda R_0}, \tag{24}$$

so that

$$\mid E_{20}\mid^2 = \left(\frac{2E_{01}A\cos\theta_i}{\lambda R_0}\right)^2. \tag{25}$$

By substituting (24) into (23) and the result of that into (21), we find the total irradiance of the light sensor to be

$$L_r = 2\sqrt{\frac{\mu}{\epsilon}}\frac{A^3}{A_s}\frac{E_{01}^2\cos^2\theta_i}{\lambda^2}\langle\mid\rho\mid^2\rangle. \tag{26}$$

For a fixed light source angle, θ_i, all the quantities except for $\langle\mid\rho\mid^2\rangle$ in (26) are constant with respect to θ_r and ϕ_r, so the shapes of the luminance curves are identical to those of the scattering coefficient for various

values of θ_r and ϕ_r. That is, for the purpose of plotting the brightness at a single surface point

as a function of the view angles we can consider the radiance to be

$$L_r = A\langle\mid\rho\mid^2\rangle, \tag{27}$$

where A is a constant.

However, if calculating the radiance at multiple points on a surface that is not planar, the light source angle changes with respect to the surface normal and (26) must be used.

The constants can be lumped to form an expression,

$$L_r = A\cos^2\theta_i\langle\mid\rho\mid^2\rangle. \tag{28}$$

5 The Lambertian Plus Specular Scattering Model

Computing the scattering coefficient is still quite complicated. Until recently, extreme simplifications were made to make direct computations of surface intensities tractable.

A Lambertian surface is an ideal matte or diffuse surface that is assumed to scatter all incident light equally in all directions into the hemisphere above the surface. This has a particularly simple radiant intensity,

$$L_r(\theta_i, \theta_r, \phi_r) = \frac{C}{\pi} \tilde{E}_1(\theta_i) \tag{29}$$

where $0 < C \leq 1$ is a constant and $\tilde{E}_1$ is the irradiance of the surface – the incident flux per unit area. (The irradiance is traditionally represented by E. But, since we are using E to represent electric field intensity, we will use $\tilde{E}$ to represent irradiance.) It is usually assumed to be constant but foreshortened by the incidence angle, θ_i, so that

$$\tilde{E}_1(\theta_i) = \tilde{E}_{10} \cos\theta_i, \tag{30}$$

Then, the radiance of a Lambertian surface is given by

$$L_r(\theta_i, \theta_r, \phi_r) = \frac{C \cdot \tilde{E}_{10}}{\pi} \cos\theta_i \tag{31}$$

Because the radiant intensity depends only on the cosine of the incidence angle, θ_i, and not the view angles, θ_r and ϕ_r, a planar patch appears equally bright from all directions. A curved surface, however, is irradiated by an amount $\tilde{E}_{10} \cos\theta_i$. If normal of the surface deviates from the illumination direction, $\cos\theta_i$ decreases. The surface is brightest at points where the surface normal is in the light-source direction. The surface appears less bright wherever $\hat{n}$ is not in direction $\hat{s}$ because less light is hitting the surface there.

A curved surface, like a sphere, with a Lambertian reflectance characteristic appears brightest at the point or points where the surface normal is in the light-source direction. (Note that at such points, the tangent plane is perpendicular to the light-source direction and is parallel to incoming planar wave fronts.) It appears less bright at points whose normals are not in the $\hat{s}$ direction. It is completely black (in the absence of ambient light) at points where $\hat{n}$ is at an obtuse angle (greater than 90°) from $\hat{s}$. For example, consider a Lambertian sphere of radius 1. Let the global coordinate origin lie at the center of the sphere. Let the view vector lie on the z-axis and the let light-source direction vector lie in the xz-plane, 45° from the z-axis in the $x < 0$ half of the plane. Then, the brightest spot on the sphere is at location $(x, y, z) = (-\sqrt{2}/2, 0, \sqrt{2}/2)$ which, not at all coincidentally, is precisely $\hat{s}$.

A perfect specular scatterer is a mirror or, equivalently, a perfectly conducting plane. A specular surface has a particularly simple radiant intensity function,

$$L_r(\theta_i, \theta_r, \phi_r) = B\,\delta(\theta_r - \theta_i)\delta(\phi_r), \tag{32}$$

where $0 < B \leq 1$ is a constant and δ is the Kronecker delta,

$$\delta(x) = \begin{cases} 0 \text{ if } x \neq 0, \\ 1 \text{ if } x = 0. \end{cases} \tag{33}$$

Thus if a smooth surface is specular, and if it is viewed from direction $\hat{k}_1$ (measured with respect to global coordinates) the light source (in direction $\hat{s}$ with respect to the global coordinates) is seen reflected in the surface at any point at which the surface normal, $\hat{n}$ bisects the angle between $\hat{k}_1$ and $\hat{s}$. So if our sphere, in the previous example were specular rather than Lambertian, the light source would appear at an angle of 22.5° from the z-axis in the $x < 0$ half of the xz-plane. That is the bright spot would be at $(x, y, z) \approx (-0.3827, 0, 0.9239)$.

In reality, there is always some blurring of the image of a light-source on a specular surface. Thus, rather than a delta function, a very narrow Gaussian is used. That is the radiant intensity is approximated by

$$L_r(\theta_i, \theta_r, \phi_r) = \frac{B}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left[\left(\frac{\theta_r - \theta_i}{\sigma}\right)^2 + \left(\frac{\phi_r}{2\sigma}\right)^2\right]}, \tag{34}$$

for some small value of σ.

The most common used model of surface reflectance has been a linear combination of Lambertian and specular radiances, equations (30) and (34). Viz,

$$L_r(\theta_i, \theta_r) = \frac{C \cdot \tilde{E}_{10}}{\pi} \cos\theta_i + \frac{B}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left[\left(\frac{\theta_r - \theta_i}{\sigma}\right)^2 + \left(\frac{\phi_r}{2\sigma}\right)^2\right]}, \tag{35}$$

This model is very easy to compute and forms the basis of many real-time 3D surface generators. However, the images so produced have a characteristic, artificial look to them. Often this is not a problem. But, given that computers are now sufficiently powerful to compute simplified versions of the Helmholtz equation (15), it is useful to examine a more accurate solution in spite of its complexity.

6 Kirchhoff-Beckmann-Spizzichino Scattering

The KBS electromagnetic scattering model is more realistic than the Lambertian plus specular model in that KBS is derived from the Helmholtz integral (15). It, however, depends on a number of possibly unrealistic assumptions [1].

1. The radius of curvature of the surface irregularities is large compared with the wavelength of incident light,

2. the reflection coefficient of the surface has unity magnitude,

3. shadowing and multiple scattering may be neglected,

4. the incident wave is plane and linearly polarized with the electrical field vector either in the plane of incidence or perpendicular to it,

5. the point of observation is sufficiently far from the surface for the scattered waves to be effectively planar, and

6. the height of the surface with respect to the average height is normally distributed with mean value zero, standard deviation σ and an autocorrelation coefficient that drops to e^{-1} over a distance T.

Let θ_i be the angle of incidence, that is, the angle from $\hat{n}$ to $\hat{s}$. Let θ_r be the polar view-angle, the angle from $\hat{n}$ to k_2. Let ϕ_r be the azimuthal view angle, the angle from the xz-plane to the plane of $\hat{n}$ and $\hat{k}_2$. In terms of these three angles vector $\vec{v}$ is

$$\vec{v} = \frac{2\pi}{\lambda}\left[(\sin\theta_i - \sin\theta_r\cos\phi_r)\hat{u}_x - \sin\theta_r\sin\phi_r\hat{u}_y - (\cos\theta_i + \cos\theta_r)\hat{u}_z\right]. \tag{36}$$

Figures 2 and 3 depict the geometry for an example surface point.

Beckmann and Spizzichino [1] derive the expected value of the square of the magnitude of the scattering coefficient, ρ, of a random rough surface from the Helmholtz equation under the assumptions listed previously. At the point (R_0, θ_r, ϕ_r),

$$\langle|\rho|^2\rangle = e^{-g}\left[\rho_0^2 + \frac{1}{\pi A}\left(\frac{\lambda T v_z}{4\cos\theta_i}\right)^2\left(\frac{|\vec{v}|}{v_z}\right)^4\sum_{m=1}^{\infty}\frac{g^m}{m!m}\exp\left(-\frac{v_{xy}^2 T^2}{4m}\right)\right], \tag{37}$$

where g is defined by

$$g = (v_z\sigma)^2 = \left[\frac{2\pi}{\lambda}\sigma(\cos\theta_i + \cos\theta_r)\right]^2. \tag{38}$$

ρ_0 is the 2D sinc function,

$$\rho_0 = \mathrm{sinc}(v_x X)\mathrm{sinc}(v_y Y), \tag{39}$$

where X and Y are the dimensions of the surface patch such that $A = 4XY$ is the area of the patch, T is the correlation length of the surface, σ is the standard deviation of the surface height distribution, and

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}, \tag{40}$$

$$v_{xy}^2 = v_x^2 + v_y^2. \tag{41}$$

Depending on the magnitude of g (38), $\langle|\rho|^2\rangle$ can be closely approximated by functions simpler than (37). When $g << 1$, the expression is called the smooth surface approximation,

$$\langle|\rho|^2\rangle = e^{-g}\left[\rho_0^2 + \frac{1}{\pi A}\left(\frac{\sigma\lambda T}{4\cos\theta_i}\right)^2|\vec{v}|^4\exp\left(-\frac{v_{xy}^2 T^2}{4}\right)\right]. \tag{42}$$

This function's behavior is dominated by its sync function component, ρ_0. If the correlation length, T, is large compared to the wavelength, λ of incident light, the second term of (42) is small compared to ρ_0, and the sync becomes very impulsive. Therefore, $\langle | \rho |^2 \rangle$ resembles a delta function which is, of course, the specular reflection function. Thus, the smooth surface approximation resembles the simple specular surface model (34). Then When $g >> 1$, (37) reduces to an expression called the rough surface approximation,

$$\langle | \rho |^2 \rangle = \frac{1}{\pi A} \left(\frac{\lambda T}{4 \sigma \cos \theta_i} \right)^2 \left(\frac{| \vec{v} |}{v_z} \right)^4 \exp \left(- \frac{v_{xy}^2 T^2}{4 v_z^2 \sigma^2} \right). \tag{43}$$

To compute the surface radiance using the KBS equations, substitute (37), (42), or (43) into (26). Figure 4 compares spheres with Kirchhoff – Beckmann - Spizzichino surfaces and a sphere with a Lambertian surface. Marks on the images indicate salient points where the surface normal of the sphere coincides with the light source direction, the specular orientation, and the view direction. Note that the Lambertian sphere is brightest at the point where light hits it most directly, whereas the Kirchhoff – Beckmann surfaces are brightest at the specular orientations.

Figure 5 plots the intensity of these spheres as a function of the angular deviation from the view vector along the line that includes the salient points.

Figure 6 shows how the reflectance curve changes for view directions out of the plane of the surface normal and light source. Notice that when the light source vector is 90° from the surface normal / view vector plane ($\theta_3 = \pi/2$), the specular reflectance direction (as determined by the location of peak reflectance) is in the same direction as the surface normal. Unfortunately, that curve is not simply an amplitude scaled and shifted version of the curve for $\theta_3 = 0$. A contour plot of the reflectance is shown in figure 7. The figure suggests that the peak value of the $\theta_3 = \pi/2$ curve is given by the value of the $\theta_3 = 0$ curve at angle $\theta_2 = 0$.

References

[1] P. Beckmann and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces*, Artech House, Norwood, MA, (1987)

[2] S. K. Nayar, K. Ikeuchi, and T. Kanade, "Surface reflection: physical and geometrical perspectives," *IEEE Trans. Pattern Anal. Machine Intell.* vol. PAMI-13, No. 7, pp. 611-634, July (1991).

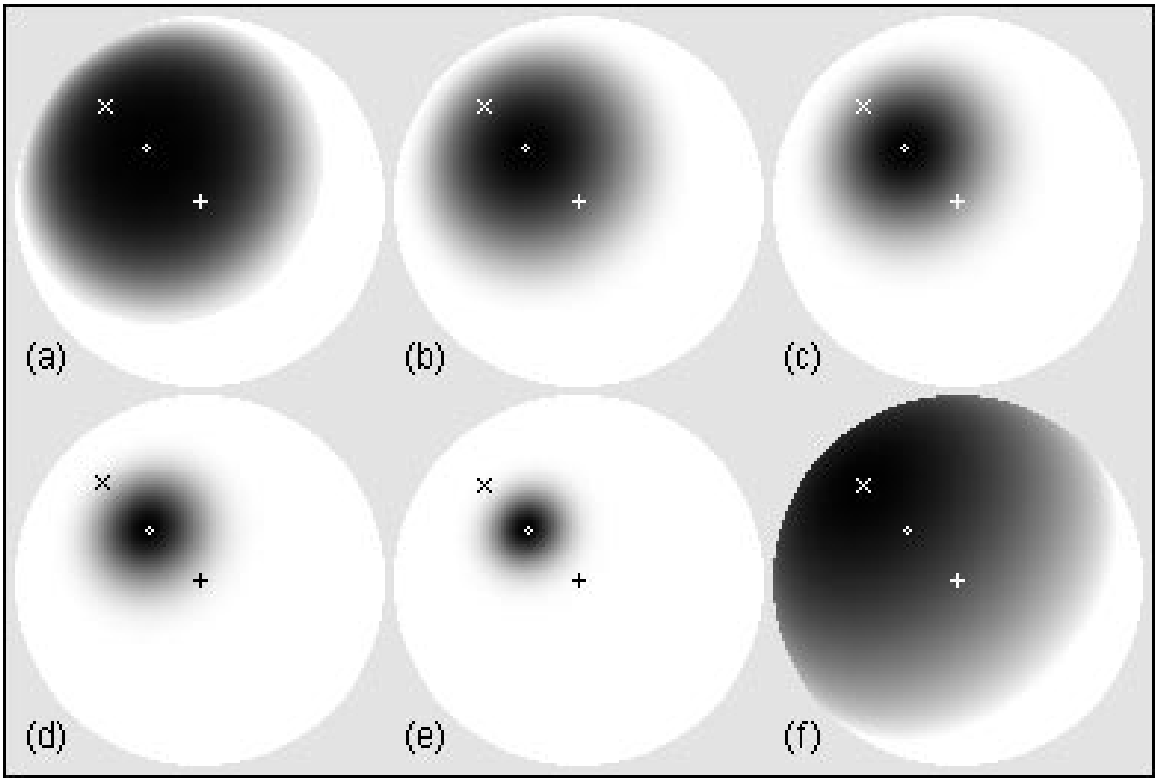

Figure 4: Examples of Kirchhoff – Beckmann surfaces, (a)-(e), and a lambertian surface (f). The objects are all spheres of equal radius. If uniformaly lit, each sphere would inscribe the frame. The images are negatives where pure black corresponds to maximum brighteness and pure white corresponds to no light. The symbols mark points on the surface where the surface normal coincides with principal directions as follows: '+' – the view direction, 'x' – the light source direction, and 'o' – the specular direction. The Kirchhoff – Beckmann parameters are (a) $T/\sigma = 3$, (b) $T/\sigma = 4$, (c) $T/\sigma = 5$, (d) $T/\sigma = 7$, (e) $T/\sigma = 10$.

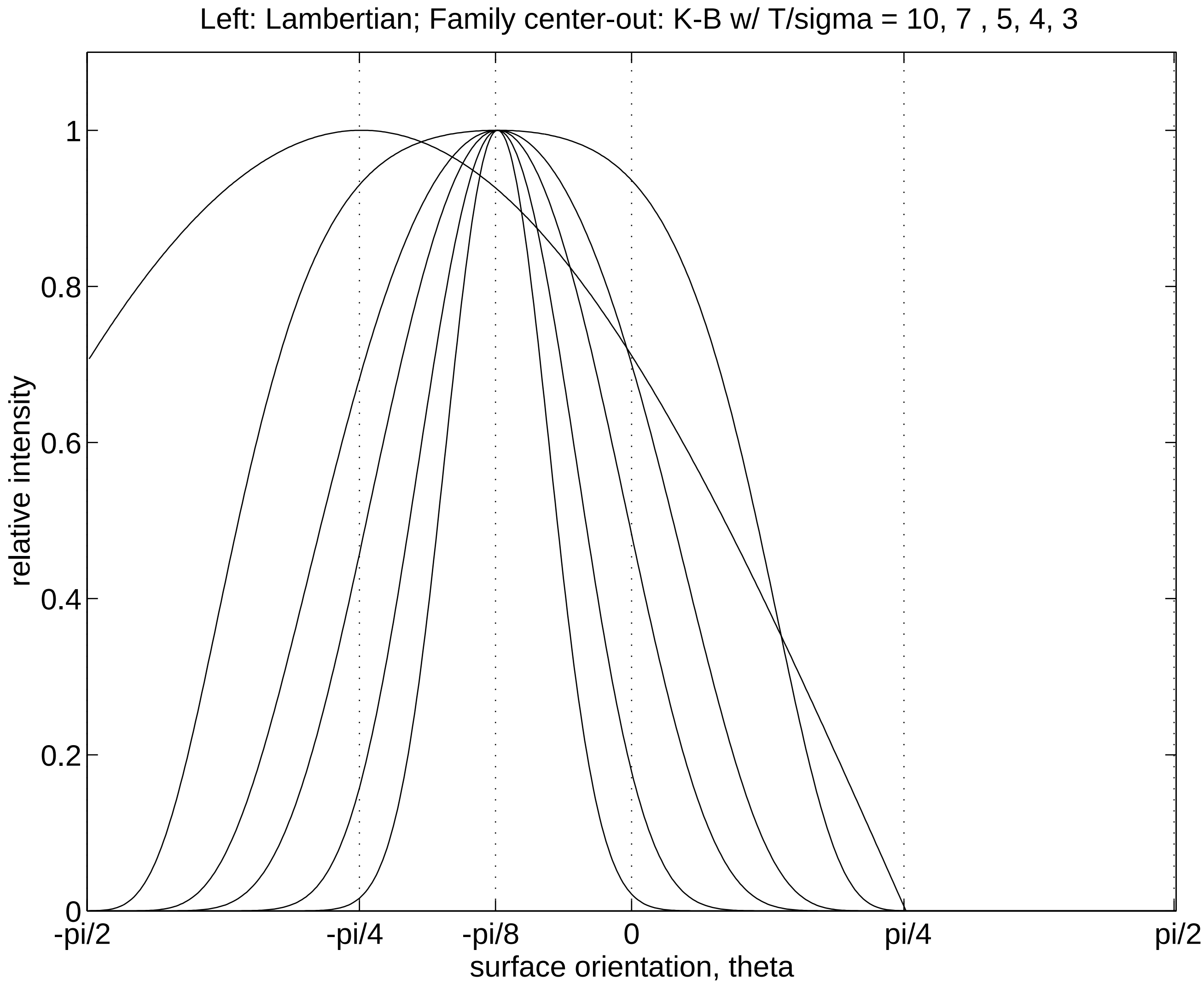

Figure 5: Intensity plots of the spheres in the previous figure. The leftmost curve is from the Lambertian surface. The family of curves are from the Kirchoff – Beckmann surfaces with parameters $T/\sigma = 10$, 7, 5, 4, 3, from the inside to outside curves. Along the plot axis, $\theta = -\pi/2$ is the upper left limb of the Lambertian sphere, $\theta = -\pi/4$ is the light source direction, $\theta = -\pi/8$ is the specular direction, $\theta = 0$, is the view direction, and $\theta = \pi/4$ is the lower right limb of the Lambertian sphere.

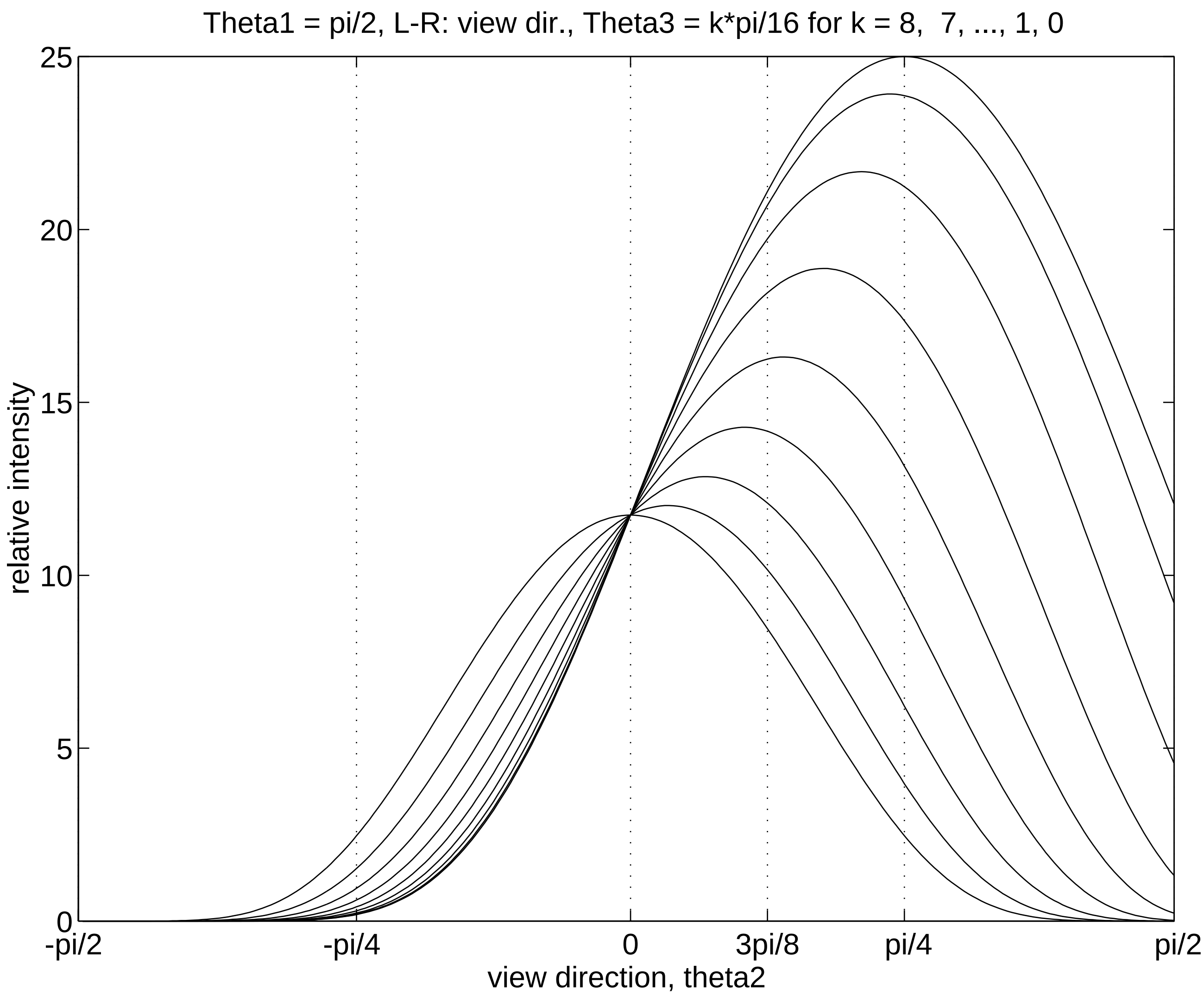

Figure 6: Reflectance of a Kirchhoff-Beckmann surface with $T/\sigma = 5$. The curves show the intensity of reflected light for a planar surface as a function of view angles θ_2 and θ_3 for $\theta_1 = 45°$. From left to right the curves are for $\theta_3 = \pi/2, 7\pi/16, 3\pi/8, 5\pi/16, \pi/4, 3\pi/16, \pi/8, \pi/16, 0$.

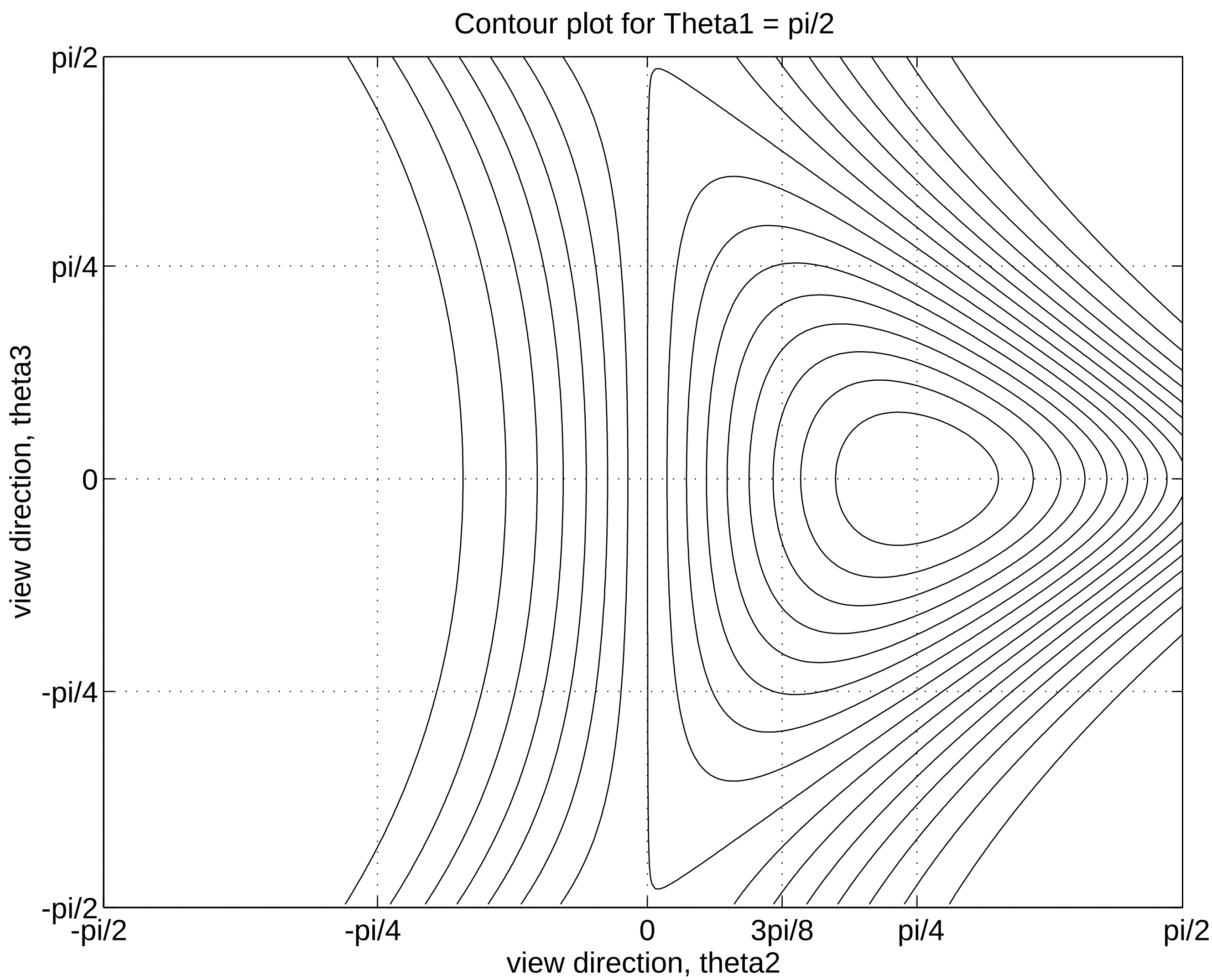

Figure 7: Contour plot of the reflectance of a Kirchhoff-Beckmann surface with $T/\sigma = 5$. The contours show points of equal intensity of reflected light for a planar surface as a function of view angles θ_2 and θ_3 for $\theta_1 = 45°$.