

Robust Differential Protection with Intermittent Cable Faults for Aircraft AC Generators

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ABSTRACT

Differential protection is a popular method to protect aircraft generators against winding faults. Traditional relay-based systems have a limited capability to distinguish between differential current resulting from a winding fault, and the one resulting from measurement noise or current saturation, resulting in false alarms and unnecessary equipment shutdown. Modern aircraft generators are monitored and controlled by advanced generator control units, and therefore, sophisticated signal processing algorithms can be implemented to enhance the differential protection performance. We propose and compare four different differential detector designs, based on the available information about measured currents, for detection of persistent, short circuit faults in the protected windings. Also, current sensors are subject to intermittent, open circuit, cable faults, resulting in degradation in the differential detection performance. We propose an optimal differential protection architecture, based on the Neyman-Pearson criterion, to detect winding short circuit faults in the presence of intermittent cable faults. In this architecture, the system switches between two different detectors, depending on the cable health state.

1 INTRODUCTION

Differential protection has been successfully used for decades in detecting generator winding faults (Breingan *et al.*, Oct 1988). It relies on the simple idea of measuring the phase current before and after the protected winding, using current transformers (CTs). If there is a discrepancy in the two measurements, then a fault is declared and the generator is shutdown as a protective measure. The zone between the two current transformers is designated as the *protected zone*, and the current transformers are designated as Differential Protection Current Transformers (DPCTs).

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Traditional differential protection relies on differential relays. This architecture is depicted in Figure 1. In normal operation, the DPCT currents are equal, and therefore, they do not operate the relay coil. When there is a winding fault (example is a short to ground), a difference current will flow through the relay coil. If the difference current exceeds a certain threshold, the relay operates and the generator is shutdown. In practice, a difference current may also be produced due to CT saturation, CT phase angle errors, or noisy measurements. Therefore, to prevent unnecessary tripping of the relay, an adaptive threshold is employed (GmbH, 2007).

Modern generator systems are monitored and controlled by a generator digital control unit (GCU). DPCT measurements are fed to the GCU, sampled at a high rate (typically $1 \mu s$) for monitoring, control, and fault detection purposes. For differential protection, typically the measurements are averaged and compared to a threshold, to decide if a winding fault exists. Because of the computational capability of modern GCU's, more sophisticated signal processing algorithms can be implemented for winding differential protection, in contrast to traditional relay-based systems. Specifically, prior knowledge about current signals could be incorporated with detection algorithms, leading to much better performance in terms of the probability of detection, P_D , and the probability of false alarm P_F . The prior knowledge can be obtained from machine real life data, or from simulation experiments.

To exploit GCU capabilities, we propose a model-based approach for winding fault differential detection. By model-based here we refer to the current *signal model*, not the generator *system model*. The generator system is a nonlinear, time-varying system, and the use of its model for fault detection is not, in general, practical (Krause *et al.*, 2002; Tantawy *et al.*, 2008). Therefore, we resort to a less-informative and more practical approach by considering the signal model. We describe and compare different optimal detector designs, with respect to the available information about the generator current signals and the noise statistical properties.

Intermittent cable faults have been reported as one of the frequent types of faults in aircraft generators. Cables are used to connect sensors and control elements to the GCU. The main intermittent cable fault

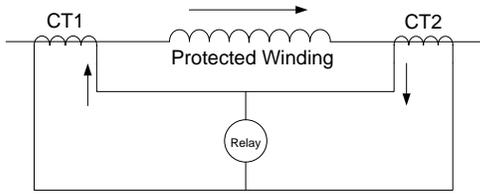


Figure 1: Traditional differential protection architecture using differential relays. Two current transformers are used to monitor the current, and the relay trips when there is a discrepancy between the two currents.

is a temporary open circuit due to loose terminal connection. An open cable results in loss of measurements transmitted to the GCU, which leads to degradation in the control performance, as well as the fault detection performance. In this paper, we limit ourselves to intermittent faults in sensor cables, specifically intermittent faults in DPCTs. Since loss of one of DPCT cables leads to a difference current, the GCU differential detection algorithm may declare a false winding fault, leading to unnecessary shutdown for the generator. If the fault profile is known, or could be estimated from the measurements, then a detector design that incorporates the fault profile parameters is expected to outperform traditional detector designs. In this paper, we present an optimal design for the differential detector in case of intermittent faults, according to Neyman-Pearson criterion of maximizing P_D for a fixed P_{FA} , assuming that the intermittent fault could be detected independently.

The rest of the paper is organized as follows: in Section 2, we propose a differential protection system architecture and formulate the research problem. Section 3 compares different detector designs, based on the available information about the machine behavior and the noise statistical properties, assuming fault-free DPCT cables. All proposed detectors are optimal in the sense that, given the information available, the detector maximizes the probability of detection given a fixed probability of false alarm (Neyman-Pearson criterion). The probability of false alarm here represents the probability of unnecessarily shutting down the generator, while the probability of detection represents the complementary probability of missing a winding fault, hence operating the machine with a faulty winding. In Section 4, we formulate the differential detection problem when there is an intermittent fault in one of the DPCT cables and provide the optimal detector design, when the intermittent fault could be detected independently. We conclude the work in Section 5.

2 STATEMENT OF THE PROBLEM

Figure 2 illustrates the proposed differential protection system architecture for a single winding. 2 DPCTs measure the current before and after the winding. Each DPCT may have an intermittent, open circuit, fault. We consider only a single fault in one of the DPCT cables (namely, CT1), since simultaneous faults in both DPCTs will produce an equal, zero current, which is easily detected by the GCU. Also, we focus our

attention on winding short circuit faults, since they represent the most dangerous type of faults on the machine operation and lifetime (Sottile *et al.*, 2006; Batzel and Swanson, 2009). Inside the GCU, an Analog to Digital converter samples the current waveforms at a much higher rate than the Nyquist rate. During a winding fault, harmonic currents are produced in the machine winding, due to the asymmetry in the magnetic field. In our current work, we work only on the fundamental harmonic, and therefore, a digital filter is added to extract the fundamental harmonic. The design of the digital filter is not treated in this paper (see e.g. (Antoniou, 2000)). The intermittent fault detector uses the sampled observations to decide if there is an intermittent cable fault. Two differential detector designs are proposed, one for the fault-free cable case, and the other one is for the cable fault case. The intermittent fault detector switches between the two detectors, based on the cable health condition.

We designate the sampled current waveform for CT1 by $i_1[n]$, and for CT2 by $i_2[n]$. The difference current is $i[n] = i_2[n] - i_1[n]$. When the machine is in normal operation (i.e., no winding faults), the current signal in both CT1 and CT2 is given by:

$$i_1[n] = i_2[n] = s_a[n] = A \cos(2\pi f_0 n + \phi_a), \quad (1)$$

where f_0 is the fundamental frequency, and ϕ_a represents the fault-free phase angle. For a balanced three phase system, the amplitude A is the same for the three phases, and the phases differ by $2\pi/3$ degrees radian.

When there is a short circuit winding fault, $i_1[n]$ and $i_2[n]$ will have different amplitudes from the normal operating value (overload condition):

$$i_1[n] = s_{\tilde{a}}[n] = \tilde{A} \cos(2\pi f_0 n + \tilde{\phi}_a) \quad (2)$$

$$i_2[n] = s_c[n] = C \cos(2\pi f_0 n + \phi_c) \quad (3)$$

Based on the above description, we can pose our research problem as follows: *Find the optimal differential protection detector design, in case of a fault-free, and faulty cables, according to Neyman-Pearson criterion of maximizing P_D for a fixed P_{FA} .*

3 DIFFERENTIAL PROTECTION: FAULT-FREE CABLES

To make the analysis tractable, we will study the discrepancy in the differential current due to only the noise effect. We assume White Gaussian Noise (WGN) process, where samples of the process are Independent and Identically Distributed (IID), according to the Gaussian distribution with zero mean and variance σ^2 .

Assuming no winding faults, the difference current will be only a noise process, i.e., $i[n] = w[n]$. When a short circuit winding fault occurs, harmonics are produced and two different currents will flow in DPCTs, with every current composed of a sum of different harmonics. Since we work only on the fundamental harmonic, we can express the differential current from Equations (2) and (3) as:

$$\begin{aligned} i[n] &= s_{\tilde{b}}[n] + w[n] = s_c[n] - s_{\tilde{a}}[n] + w[n] \\ &= \tilde{B} \cos(2\pi f_0 n + \tilde{\phi}_b) + w[n], \end{aligned} \quad (4)$$

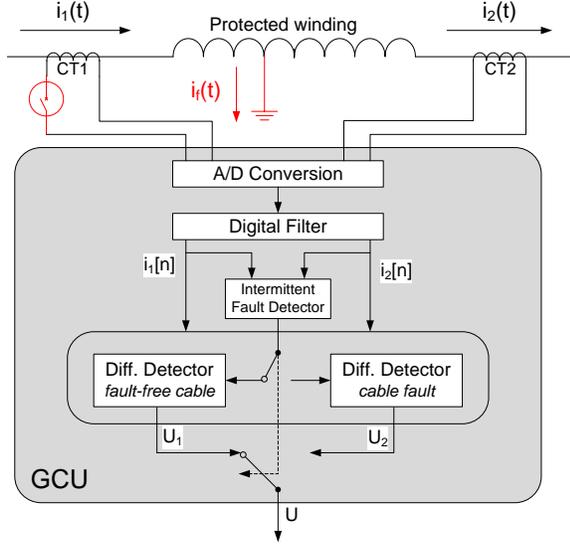


Figure 2: Differential protection architecture. Two current transformers are used to monitor the current and protect the winding.

where

$$\tilde{B} = \sqrt{\tilde{A}^2 + C^2 + 2\tilde{A}C \cos(\tilde{\phi}_a + \phi_c)}$$

$$\tilde{\phi}_b = \arctan \frac{C \sin(\phi_c) - \tilde{A} \sin(\tilde{\phi}_a)}{C \cos(\phi_c) + \tilde{A} \cos(\tilde{\phi}_a)}$$

The amplitude \tilde{B} and the phase $\tilde{\phi}_b$ depend on the magnitude and location of the winding fault, and they are not known a priori. Without prior assumptions about the fault nature, not much can be said about \tilde{B} and $\tilde{\phi}_b$, except that $\tilde{A} - C \leq \tilde{B} \leq \tilde{A} + C$, which holds directly from the inequality $-1 \leq \cos(x) \leq 1$. If we assume that the winding fault is an almost complete short circuit, then C is very small and we can assume $\tilde{B} \approx \tilde{A}$, without knowledge about the phase angles. If, however, C cannot be neglected, which is the case when there is a partial short circuit, then \tilde{B} varies widely depending on the phase angle values, and further assumptions have to be defined, to accurately quantify the detection performance.

The above description represents the classical signal detection problem in WGN, with the signal being the difference current. This problem could be stated formally as the following hypothesis testing problem:

$$\mathcal{H}_0 : i[n] = w[n]$$

$$\mathcal{H}_1 : i[n] = \tilde{B} \cos(2\pi f_0 n + \tilde{\phi}_b) + w[n]$$

The optimal detector for this hypothesis testing problem depends on our knowledge about the sinusoidal signal. Perfect knowledge will produce the best performance. However, as mentioned previously, the magnitude and phase are usually hard to know a priori, since they depend on the fault type, unless the fault information is known from real life data or simulation experiments. Therefore, because the signal parameters are not completely known, we have to accept a

performance loss in the detectors. In the following discussion, we compare different detectors, based on the signal knowledge available. The results are drawn directly from classical signal detection theory ((Kay, 1998)).

In the following discussion, we assume that the decision is based on the N -dimensional random vector $\mathbf{i} = [i[0] \ i[1] \ \dots \ i[N-1]]$. To simplify the notation, we assume that the data samples start at the time instant $n = 0$. The detection for signals with an unknown delay, n_0 , can be treated similarly by estimating n_0 from the data samples, with a slight loss in the detection performance. The interested reader should refer to (Kay, 1998). The design of the detector is the determination of the test statistic $T(\mathbf{i})$, as a function of the observation vector \mathbf{i} , and the detector threshold γ :

$$T(\mathbf{i}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \quad (5)$$

3.1 Sample Average Detector

In this case, no signal knowledge is used, and the detector is given by:

$$T(\mathbf{i}) = \sum_{n=0}^{N-1} i[n] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \quad (6)$$

It can be easily shown that the detector Receiver Operating Characteristic (ROC) curve is given by (Kay, 1998):

$$P_D = Q \left(Q^{-1}(P_F) - \sqrt{\frac{\mathcal{E}_1}{\sigma^2}} \right), \quad (7)$$

where:

$$\mathcal{E}_1 = \frac{1}{N} \left[\sum_{n=0}^{N-1} s_b[n] \right]^2 = \frac{\tilde{B}^2}{N} \left[\sum_{n=0}^{N-1} \cos(2\pi f_0 n + \tilde{\phi}_b) \right]^2 \quad (8)$$

and $Q(\cdot)$ is the right-tail probability for the standard normal PDF. Since the signal amplitude, \tilde{B} , is a measure of the fault magnitude, one way to express the detection performance is in terms of the signal to noise ratio, \tilde{B}/σ . From Equations (7) and (8) we get:

$$P_D = Q \left(Q^{-1}(P_F) - \left(\frac{\tilde{B}}{\sigma} \right) \frac{\sum_{n=0}^{N-1} \cos(2\pi f_0 n + \tilde{\phi}_b)}{\sqrt{N}} \right) \quad (9)$$

3.2 Known Sinusoidal Signal Detector

If $s_b[n]$ is completely known, then the optimal detector is the matched filter, given by:

$$T(\mathbf{i}) = \sum_{n=0}^{N-1} i[n] s_b[n] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \quad (10)$$

and the ROC curve is given by:

$$P_D = Q \left(Q^{-1}(P_F) - \sqrt{\frac{\mathcal{E}_2}{\sigma^2}} \right), \quad (11)$$

where:

$$\mathcal{E}_2 = \sum_{n=0}^{N-1} s_b^2[n] = \tilde{B}^2 \sum_{n=0}^{N-1} \cos^2(2\pi f_0 n + \tilde{\phi}_b) \quad (12)$$

Accordingly:

$$P_D = Q \left(Q^{-1}(P_F) - \left(\frac{\tilde{B}}{\sigma} \right) \sqrt{\sum_{n=0}^{N-1} \cos^2(2\pi f_0 n + \tilde{\phi}_b)} \right) \quad (1)$$

3.3 Unknown Amplitude Detector

If the signal amplitude is not known, the samples are used to calculate the Maximum Likelihood Estimator (MLE) for the amplitude, and the detector is given by:

$$T(\mathbf{i}) = \left[\sum_{n=0}^{N-1} i[n] \cos(2\pi f_0 n + \tilde{\phi}_b) \right]^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \quad (14)$$

and the ROC curve is given by:

$$\begin{aligned} P_D &= Q \left(Q^{-1}(P_F/2) - \sqrt{\frac{\mathcal{E}_2}{\sigma^2}} \right) + Q \left(Q^{-1}(P_F/2) + \sqrt{\frac{\mathcal{E}_2}{\sigma^2}} \right) \\ &= Q \left(Q^{-1}(P_F/2) - \left(\frac{\tilde{B}}{\sigma} \right) \sqrt{\sum_{n=0}^{N-1} \cos^2(2\pi f_0 n + \tilde{\phi}_b)} \right) \\ &+ Q \left(Q^{-1}(P_F/2) + \left(\frac{\tilde{B}}{\sigma} \right) \sqrt{\sum_{n=0}^{N-1} \cos^2(2\pi f_0 n + \tilde{\phi}_b)} \right) \end{aligned} \quad (15)$$

where \mathcal{E}_2 is given by Equation (12). The performance of this detector can be easily shown to be worse than the matched filter.

3.4 Unknown Amplitude and Phase Angle Detector

In practice, when a fault occurs, the resulting sinusoidal signal has unknown amplitude and phase angle. Therefore, this case represents the practical situation with most winding faults, and its performance should be compared with the sample average detector, where no signal knowledge is exploited. The unknown amplitude, \tilde{B} , and the phase angle, $\tilde{\phi}_b$, could be estimated from the sample data, using MLE, and the detector is given by:

$$\begin{aligned} T(\mathbf{i}) &= \frac{1}{N} \left[\left(\sum_{n=0}^{N-1} i[n] \cos(2\pi f_0 n) \right)^2 \right. \\ &\left. + \left(\sum_{n=0}^{N-1} i[n] \sin(2\pi f_0 n) \right)^2 \right] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \end{aligned} \quad (16)$$

and the ROC curve is given by:

$$P_D = Q_{\chi^2_2}(\tilde{\lambda}) \left(2 \ln \frac{1}{P_F} \right), \quad (17)$$

where $\tilde{\lambda} = N\tilde{B}^2/2\sigma^2$, and $Q_{\chi^2_2}(\tilde{\lambda})$ is the right tail probability for the non-central chi-squared PDF with 2 degrees of freedom, and non-centrality parameter $\tilde{\lambda}$.

3.5 Performance Comparison

Figure 3 illustrates the detection performance for the four detectors presented in Sections 3.1 to 3.4. The probability of detection is plotted against the signal to

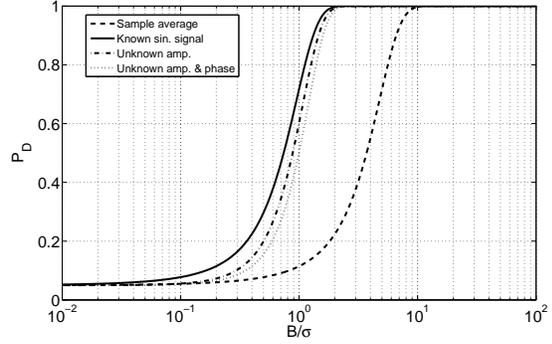


Figure 3: Detection performance for the four detectors in Section 3. The sample average detector has the worst performance, since it does not use the available signal information. Slight degradation is noted when the amplitude and phase of the sinusoidal signal are not known.

noise ratio \tilde{B}/σ (using a semilog scale), using the parameters in Table 1. As expected, the best performance is achieved by the matched filter detector, which assumes complete knowledge about the sinusoidal signal. The detector with unknown amplitude has a slight degradation in the performance, and the loss of performance is not significant when, additionally, the phase is not known.

The sample average detector has the worst performance, and not comparable to the other three detectors. This is because the detector does not use any signal information. Traditional differential protection systems rely on this type of detectors, and therefore, using the detector with unknown amplitude and phase outperforms it by a large magnitude. For example, from Figure 3, at signal to noise ratio = 1, the sample average detector achieves $P_D \approx 0.115$, while the detector with unknown amplitude and phase achieves $P_D \approx 0.5$. The advantage of the later detector is that it requires only knowledge about the frequency of the sinusoidal signal.

Table 1: Parameter values used for evaluating the detection performance of the four detectors presented in Section 3. The performance is plotted in Figure 3.

Parameter	Value
N	10
f_0	1/7
$\tilde{\phi}_b$	0
P_F	0.05

4 DIFFERENTIAL PROTECTION: INTERMITTENT CABLE FAULTS

In this section, we consider the case of an intermittent cable fault in one of the CT cables. We assume the fault occurs in CT1 (refer to Figure 2), and the other case could be treated similarly. The intermittent fault is an open circuit condition, where the measured current

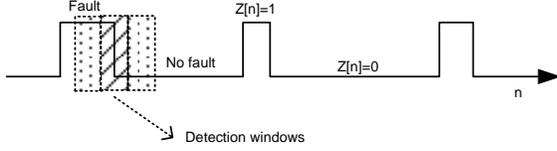


Figure 4: Intermittent cable fault model. The detection windows size is much smaller than the fault time. The shaded detection window covers both faulty and fault-free samples, and its effect on the overall performance is neglected.

by the GCU becomes zero, instead of its normal value as a sample from a sinusoidal signal.

4.1 Fault Model

We represent the intermittent fault by the discrete-time, binary, random process $Z[n]$, defined as:

$$Z[n] = \begin{cases} 0 & \text{no fault} \\ 1 & \text{cable fault} \end{cases} \quad (18)$$

We have four different cases: 1) no cable fault and no winding fault, the measured current difference is only the measurement noise. 2) cable fault and no winding fault, the current difference is between the normal sinusoidal signal of the generator, $s_a[n]$, (see Equation (1)), received from CT2, and the zero current received from CT1. The sinusoidal signal in this case is the normal signal of the generator, $s_a[n]$. Therefore, its magnitude and phase can be assumed known. 3) no cable fault and winding fault, the current difference is $s_{\tilde{b}}[n]$, defined by Equation (4). 4) cable fault and winding fault, the current difference is between an unknown sinusoidal signal from CT2, $s_c[n]$, defined by Equation (3), and the zero current from CT1.

We assume that during any detection window of size N , the random process is either in state 0 or state 1, but not both, i.e., $Z[n] = 0$ or $Z[n] = 1$. This assumption is justified by the fast sampling rate of the GCU, and the relatively slow dynamics of the intermittent fault. Figure 4 depicts this scenario, where the time scale is divided into small detection windows. The case where the detection window covers both faulty and fault-free samples will have a very small effect on the overall detection performance, and therefore, could be neglected.

4.2 Detector Design

The detection of an open-circuit intermittent cable fault can be achieved by a zero-crossing detector for the sampled current waveform from CT1. The details of the intermittent fault detector are not in the scope of this paper. Rather, we assume that an intermittent fault is detected successfully. Using the above-mentioned fault model, the four cases mentioned in Section 4.1 could be split into the following *two* hypothesis testing problems:

$$Z[n] = 0 : i[n] = \begin{cases} w[n] & \mathcal{H}_0 \\ \bar{B} \cos(2\pi f_0 n + \bar{\phi}_b) + w[n] & \mathcal{H}_1 \end{cases} \quad (19)$$

$$Z[n] = 1 : i[n] = \begin{cases} A \cos(2\pi f_0 n + \phi_a) + w[n] & \mathcal{H}_0 \\ C \cos(2\pi f_0 n + \phi_c) + w[n] & \mathcal{H}_1 \end{cases} \quad (20)$$

The overall performance of the DP detector is the average performance of the two detectors for the hypothesis testing problems in Equations (19) and (20). Therefore, to maximize the overall performance, we maximize the individual detectors performance.

Equation (19) represents the detection problem discussed in Section 3.4, and the detector performance is given by Equation (17). The detection problem in Equation (20) is to detect one of two sinusoidal signals, one with known amplitude and phase, and another with unknown amplitude and phase. If we defined $i'[n] = i[n] - s_a[n]$, then we obtain an equivalent hypothesis testing problem for $i'[n]$. This can be easily shown by noting that the distributions $p_I(i; \mathcal{H}_k)$, $k = \{0, 1\}$ are only shifted when subtracting a deterministic quantity. Therefore, the likelihood ratio is the same, and hence, the detector performance is the same. Using Equations (1) and (3), we define:

$$\begin{aligned} s_b[n] &= s_c[n] - s_a[n] \\ &= B \cos(2\pi f_0 n + \phi_b), \end{aligned} \quad (21)$$

where:

$$\begin{aligned} B &= \sqrt{A^2 + C^2 + 2AC \cos(\phi_a + \phi_c)} \\ \phi_b &= \arctan \frac{C \sin(\phi_c) - A \sin(\phi_a)}{C \cos(\phi_c) + A \cos(\phi_a)} \end{aligned}$$

Accordingly, we obtain the following equivalent hypothesis testing problem:

$$i'[n] = \begin{cases} w[n] & \mathcal{H}_0 \\ B \cos(2\pi f_0 n + \phi_b) + w[n] & \mathcal{H}_1 \end{cases} \quad (22)$$

This problem is identical to the one in Equation (19), and therefore, the detector design is given by Equation (16), by replacing $i[n]$ by $i'[n]$:

$$\begin{aligned} T(i) &= \left(\sum_{n=0}^{N-1} [i[n] - A \cos(2\pi f_0 n + \phi_0)] \cos(2\pi f_0 n) \right)^2 \\ &+ \left(\sum_{n=0}^{N-1} [i[n] - A \cos(2\pi f_0 n + \phi_0)] \sin(2\pi f_0 n) \right)^2 \end{aligned} \quad (23)$$

The detector performance is given by Equation (17), with the non-centrality parameter $\lambda = NB^2/2\sigma^2$.

The same comment for signal $s_{\tilde{b}}[n]$ is applied here for $s_b[n]$, where $A - C \leq B \leq A + C$. For the case of an almost complete short circuit, $C \approx 0$, and $B \approx A$, regardless of the phase information. It should be highlighted that in this later case, $B \leq \bar{B}$, since when a short circuit fault occurs, the machine is overloaded and $A < \bar{A}$ (refer to Equation (4)). This has the implication that the differential detection performance with a fault-free cable is better than the case with an intermittent cable fault, since the detector performance increases with increasing the detected signal magnitude.

4.3 Optimal Detection Performance

We now have two differential detectors, one for the fault-free cable case (Equation (16)), and another detector for the faulty cable case (Equation (23)). If we assume that the probability of the cable being in state 0

(no fault) is π_0 , and the probability of being in state 1 (open circuit fault) is $\pi_1 = 1 - \pi_0$, then we can express the overall performance by:

$$E[P_D] = \pi_0 Q_{\chi^2_2}(\tilde{\lambda}) \left(2 \ln \frac{1}{P_{F_1}} \right) + (1 - \pi_0) Q_{\chi^2_2}(\lambda) \left(2 \ln \frac{1}{P_{F_2}} \right) \quad (24)$$

$$E[P_F] = \pi_0 P_{F_1} + (1 - \pi_0) P_{F_2} \quad (25)$$

for every pair of P_{F_1} and P_{F_2} . This new ROC curve represents an average of the two ROC curves for the fault-free cable and faulty cable cases, with π_0 and π_1 as the weighting factors.

To design the two detectors, we need to specify their respective thresholds. In (Tantawy *et al.*, 2009), it is proved that the optimal detection that maximizes $E[P_D]$ with the constraint $E[P_F] = \alpha$ occurs when all individual detectors have the same likelihood ratio threshold, γ , but for the detector in Equation (17), we have (see (Kay, 1998)):

$$P_F = \frac{1}{\gamma} \quad (26)$$

Therefore, by using Equation (25), we have the result:

$$P_{F_1} = P_{F_2} = \frac{1}{\gamma} = \alpha \quad (27)$$

where α is the constraint on $E[P_F] = \alpha$. Accordingly, to maximize $E[P_D]$ with the constraint $E[P_F] = \alpha$, we select the individual operating points for the two ROC curves as $\left(\alpha, Q_{\chi^2_2}(\tilde{\lambda}) \left(2 \ln \frac{1}{\alpha} \right) \right)$ and $\left(\alpha, Q_{\chi^2_2}(\lambda) \left(2 \ln \frac{1}{\alpha} \right) \right)$, for the fault-free and faulty cable detectors, respectively. The performance is then given by:

$$E[P_D] = \pi_0 Q_{\chi^2_2}(\tilde{\lambda}) \left(2 \ln \frac{1}{\alpha} \right) + (1 - \pi_0) Q_{\chi^2_2}(\lambda) \left(2 \ln \frac{1}{\alpha} \right) \quad (28)$$

$$E[P_F] = \alpha \quad (29)$$

It should be highlighted that for the proposed detector, the operating point does not depend on the fault profile parameters π_0 and π_1 . This is not true in general (see (Tantawy *et al.*, 2009)), and it may be necessary to estimate the fault model parameters online, and adapt individual detector thresholds accordingly.

Figure 5 is a 3D plot for Equation (24), using the parameters shown in Table 2. From the figure, it is clear that for all combinations of P_{F_1} and P_{F_2} , $E[P_D]$ is maximized along the line $P_{F_1} = P_{F_2}$, which coincides with the result obtained in Equation (27).

Figure 6 illustrates the ROC curves for the two differential detectors. The curves are direct plots for Equation (17), using the parameters in Table 2. Also, the average optimal performance (Equation 28) for different values of α is shown. The optimal average performance is bounded by the individual detectors performance.

Using Equation (27), we can also represent the detection performance for a fixed $P_F = \alpha$, as a function

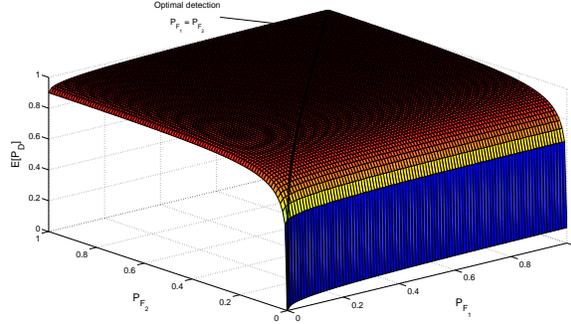


Figure 5: 3D plot for Equation (24), representing the average detection performance for the differential detectors with and without intermittent cable faults. Optimal detection is marked along the line $P_{F_1} = P_{F_2}$.

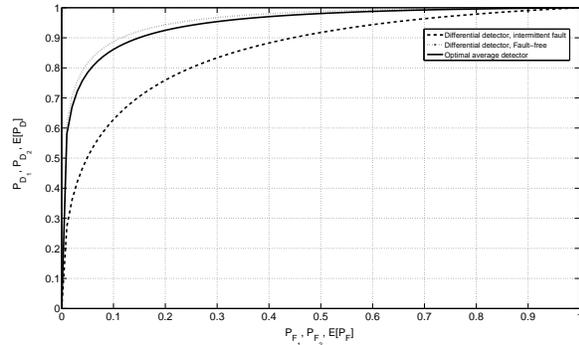


Figure 6: ROC curves for differential detectors, with and without intermittent cable faults. The average optimal detector is bounded by the two ROC curves and represents the best performance obtained given the fault profile.

Table 2: Parameter values used for evaluating the average detection performance for the differential detectors with and without intermittent cable faults. The performance is plotted in Figures 5,6, and 7

Parameter	Value
N	10
f_0	1/7
\bar{B}/σ	1
B/σ	2
π_0	0.1

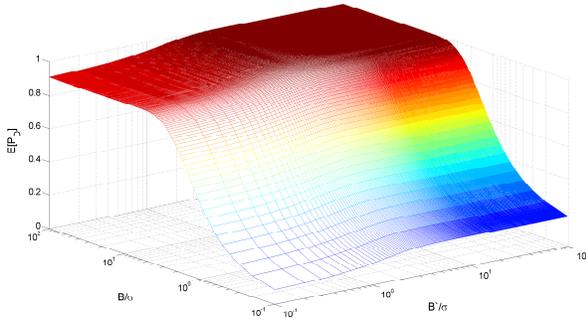


Figure 7: Performance of the optimal average detector versus individual signal to noise ratios. The plot could be used to quantify the performance for different faults, and accordingly, different signal to noise ratios.

of individual signal to noise ratios. This is a 3D plot for Equation (28) as a function of \bar{B}/σ and B/σ , using the parameters in Table 2, and $E[P_F] = \alpha$. This plot is useful in specifying the performance with respect to different winding faults, if a mapping could be constructed between winding faults and signal to noise ratios.

5 CONCLUSION AND FUTURE WORK

Differential protection performance can be enhanced by utilizing prior information about the measured current signals. Information about the sinusoidal signal frequency can enhance the performance by orders of magnitude, compared to the traditional sample average detector. Information about the magnitude and phase angle of the measured signal cannot be obtained a priori, since they depend on the fault magnitude and location, and their effect on the detection performance is slight.

With intermittent, open circuit, current sensor cable fault, two detectors have to be designed for the fault-free and faulty cable case. The optimal detection, according to Neyman-Pearson formulation, occurs when the individual detectors have the same probability of false alarm, which is set equal to the desired global constraint on the expectation of the probability of false alarm. The optimal detection operating point does not depend on the fault profile, and therefore, online estimation for the fault probabilities is not required. If a mapping could be established between different types

of faults and the corresponding generated sinusoidal wave in the generator windings, then detection performance can be quantified for every fault type. This mapping could be constructed by simulation studies and from real-life data. We will pursue this direction in future work. Also, more sophisticated intermittent cable faults, and a comparison between detection performance with these models and the cable fault model presented here, are currently being investigated.

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