consensus algorithms designed to withstand outliers [9, 13]. The problem of robust consensus to outliers does not assume a threat model, such as malicious or Byzantine nodes. Instead, some measurements may be statistical outliers caused by noisy measurements and the goal is to reach consensus on the measurements in a manner that reduces the error introduced by the outliers. In these works the nodes with outlier measurements are cooperative in the consensus process. Therefore, such techniques are not designed to work in the presence of adversaries.

7. CONCLUSIONS

In this paper, we have studied a low complexity protocol (algorithm), ARC-P, for reaching consensus in networked multi-agent systems with adversaries. We formulated a consensus problem, the adversarial asymptotic agreement problem, appropriate for distributed control applications. We defined two different models for adversaries depending on how information is conveyed. Malicious agents must convey the same information to each neighbor, whereas Byzantine agents may convey different information to each neighbor. We analyzed the convergence properties of ARC-P in directed networks with fixed and switching topologies in the presence of malicious and Byzantine agents, while restricting the range of the switching signal so that each topology satisfies sufficient conditions on the in-degrees and out-degrees of nodes in the network. Finally, we examined the conservativeness of the conditions.

Based on the examples in Section 5, it is clear that traditional graph theoretic metrics like minimum degree and connectivity are not suitable for characterizing under which conditions ARC-P ensures agreement. Therefore, to ascertain conditions which are both necessary and sufficient, new graph theoretic metrics are needed.

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9. REFERENCES

[1] D. Dolev. The Byzantine generals strike again. *Journal of Algorithms*, 3(1):14 – 30, 1982.

[2] D. Dolev, N. A. Lynch, S. S. Pinter, E. W. Stark, and W. E. Weihl. Reaching approximate agreement in the presence of faults. *Journal of the ACM*, 33(3):499 – 516, 1986.

[3] J. A. Fax and R. M. Murray. Information flow and cooperative control of vehicle formations. *IEEE Trans. on Aut. Control*, 49(9):1465 – 1476, 2004.

[4] D. Geller and F. Harary. Connectivity in digraphs. In *Recent Trends in Graph Theory*, volume 186 of *Lect. Notes in Math.*, pages 105–115. Springer Berlin / Heidelberg, 1971.

[5] V. Gupta, C. Langbort, and R. Murray. On the robustness of distributed algorithms. In *IEEE Conf. on Decision and Control*, Dec. 2006.

[6] T. T. Johnson and S. Mitra. Safe flocking in spite of actuator faults using directional failure detectors. *Journal of Nonlinear Sys. and App.*, 2(1-2):73–95, 2011.

[7] L. Lamport, R. Shostak, and M. Pease. The Byzantine generals problem. *ACM Trans. Program. Lang. Syst.*, 4(3):382–401, 1982.

[8] H. J. LeBlanc and X. D. Koutsoukos. Consensus in networked multi-agent systems with adversaries. In *Proc. of the 14th Int. Conf. on Hybrid systems: Computation and Control*, (HSCC '11), pages 281–290, Chicago, IL, 2011.

[9] J. Li, E. Elhamifar, I.-J. Wang, and R. Vidal. Consensus with robustness to outliers via distributed optimization. In *IEEE Conf. on Decision and Control*, pages 2111–2117, Dec. 2010.

[10] D. Liberzon. *Switching in Systems and Control*. Birkhauser, Boston, MA, USA, 2003.

[11] N. A. Lynch. *Distributed Algorithms*. Morgan Kaufmann Publishers Inc., San Francisco, California, 1997.

[12] M. Mesbahi and M. Egerstedt. *Graph Theoretic Methods in Multiagent Networks*. Princeton University Press, Princeton, New Jersey, 2010.

[13] E. Montijano, S. Martínez, and S. Sagués. De-RANSAC: robust distributed consensus in sensor networks. *IEEE Transactions on Systems, Man, and Cybernetics: part B*. Submitted, May 2010.

[14] R. Olfati-Saber, J. A. Fax, and R. M. Murray. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(1):215–233, 2007.

[15] R. Olfati-Saber, E. Franco, E. Frazzoli, and J. Shamma. Belief consensus and distributed hypothesis testing in sensor networks. In *Networked Embedded Sensing and Control*, volume 331 of *Lecture Notes in Control and Information Sciences*, pages 169–182. Springer Berlin / Heidelberg, 2006.

[16] F. Pasqualetti, A. Bicchi, and F. Bullo. Distributed intrusion detection for secure consensus computations. In *IEEE Conf. on Decision and Control*, pages 5594 –5599, Dec. 2007.

[17] F. Pasqualetti, A. Bicchi, and F. Bullo. On the security of linear consensus networks. In *IEEE Conf. on Decision and Control*, pages 4894 – 4901, Dec. 2009.

[18] F. Pasqualetti, A. Bicchi, and F. Bullo. Consensus computation in unreliable networks: A system theoretic approach. *IEEE Trans. on Aut. Control*, 57(1):90–104, Jan. 2012.

[19] F. Pasqualetti, R. Carli, A. Bicchi, and F. Bullo. Identifying cyber attacks under local model information. In *IEEE Conf. on Decision and Control*, pages 5961–5966, Dec. 2010.

[20] M. Pease, R. Shostak, and L. Lamport. Reaching agreement in the presence of faults. *J. ACM*, 27(2):228–234, 1980.

[21] N. Rouche, P. Habets, and M. Laloy. *Stability Theory by Liapunov's Direct Method*, volume 22 of *Applied Mathematical Sciences*. Springer-Verlag, New York, 1977.

[22] S. Sundaram and C. Hadjicostis. Distributed function calculation via linear iterations in the presence of malicious agents; part I: Attacking the network. In *American Control Conf.*, pages 1350 –1355, June 2008.

[23] S. Sundaram and C. Hadjicostis. Distributed function calculation via linear iterations in the presence of malicious agents; part II: Overcoming malicious behavior. In *American Control Conf.*, pages 1356 –1361, June 2008.

[24] S. Sundaram and C. Hadjicostis. Distributed function calculation via linear iterative strategies in the presence of malicious agents. *IEEE Trans. on Aut. Control*, 56(7):1495 –1508, July 2011.

[25] A. Teixeira, H. Sandberg, and K.H. Johansson. Networked control systems under cyber attacks with applications to power networks. In *American Control Conf.*, pages 3690–3696, July 2010.

[26] J. N. Tsitsiklis. *Problems in Decentralized Decision Making and Computation*. PhD thesis, Dept of EECS, MIT, 1984.