

# A Cross-Layer Design for Decentralized Detection in Tree Sensor Networks

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**Abstract**—The design of wireless sensor networks for detection applications is a challenging task. On one hand, classical work on decentralized detection does not consider practical wireless sensor networks. On the other hand, practical sensor network design approaches that treat the signal processing and communication aspects of the sensor network separately result in suboptimal detection performance because network resources are not allocated efficiently. In this work, we attempt to cross the gap between theoretical decentralized detection work and practical sensor network implementations. We consider a cross-layer approach, where the quality of information, channel state information, and residual energy information are included in the design process of tree-topology sensor networks. The design objective is to specify which sensors should contribute to a given detection task, and to calculate the relevant communication parameters. We compare two design schemes: (1) *direct transmission*, where raw data are transmitted to the fusion center without compression, and (2) *in-network processing*, where data is quantized before transmission. For both schemes, we design the optimal transmission control policy that coordinates the communication between sensor nodes and the fusion center. We show the performance improvement for the proposed design schemes over the classical decoupled and maximum throughput design approaches.

**Keywords:** Decentralized detection, networked information fusion, wireless sensor network, transmission control policy, optimization.

## I. INTRODUCTION

The deployment of Wireless Sensor Networks (WSNs) in decentralized detection applications is motivated by the availability of low cost sensors, combined with the advances in communication network technologies. In Decentralized Detection (DD), multiple sensors collaborate to distinguish between two or more hypotheses. In many practical applications, sensors are distributed geographically and connected in a tree configuration to sample the environment, pre-process the data, and communicate the information to the fusion center for final decision-making.

The deployment of WSNs in detection applications brings new challenges, in addition to the design of signal processing algorithms at the application layer that has been extensively addressed, e.g. [1], [2]. Protocols for communication layers have to be co-designed to optimize the detection performance. The layered approach commonly adopted to design wireless

networks may not be appropriate for detection applications, as it neither provides the optimal resource allocation nor exploits the application domain knowledge. A cross-layer design approach is desired for efficient implementation of WSNs in decentralized detection applications.

The cross-layer design approach has been recently explored for the design of Media Access Control (MAC) protocols for parallel topology (direct transmission) sensor networks in detection applications. Decision fusion over slotted ALOHA MAC employing a collision resolution algorithm is studied in [3]. A thorough investigation of the design of MAC transmission policies to minimize the error probability has been considered in [4], where sensors are assumed non-identical, and the MAC policy is assumed stochastic. The cross-layer approach is also considered in [5] where an integrated model for the physical channel and the queuing behavior for sensors is developed. For tree networks, energy-efficient routing for signal detection in WSNs is considered in [6], where the objective is to find the optimal route for local data from a target location to the fusion center, in order to maximize the detection performance or to minimize the energy consumption. Cooperative routing for distributed detection in large sensor networks is studied in [7] using a link metric that characterizes the detection error exponent. Optimal communication rate allocation for multihop sensor networks deployed for DD is studied in [8], where no medium access contention is assumed. For a survey on the interplay between signal processing and networking in sensor networks, see [9] and the references therein.

Our work is different in two main aspects: (1) we integrate the physical layer, MAC layer, and the detection application layer in one unified system model, and (2) we include the three quality measures that were previously treated separately, namely the Quality of Information (QoI), Channel State Information (CSI), and Residual Energy Information (REI) for each sensor. We considered the Transmission Control Policy (TCP) design for parallel topology sensor networks in [10]. In this paper, we design the optimal TCP that coordinates the communication between sensor nodes connected in a tree configuration. Our approach formulates the detection performance measure as a function of the parameters of the integrated system model. We then solve a constrained optimization problem to obtain the TCP variables that maximize the detection performance. We have the following design assumptions: (1) *Minimal movement*

of sensor nodes. This assumption allows us to consider the large scale fading component only for the physical channel, hence simplifying the analysis. (2) *Slotted ALOHA MAC*. The traditional assumption of a dedicated orthogonal channel between each sensor node and its parent node may not be feasible in practice. Slotted ALOHA multiaccess scheme, on the other hand, has been successfully deployed in practice. We use a simplified version of the slotted ALOHA protocol, ignoring the protocol specifics, to keep the analysis tractable. (3) *Synchronization*. We assume that sensors are synchronized, which allows us to model the network as a discrete time system, hence simplifying the analysis. (4) *Transmission scheme*. We assume two transmission schemes, direct transmission where raw observations are sent directly to the fusion center without local processing, and in-network processing where information is compressed and quantized locally before transmission.

We summarize the contributions of our work as follows: (1) **Integrated model for the detection system**. The model captures the physical channel, MAC protocol, and the detection application models, and their interactions. The model also incorporates the QoI, CSI, and REI measures for each sensor. (2) **Design of a complete transmission control policy**. We design the TCP for the tree topology for a finite number of sensors, rather than asymptotically. The TCP variables include retransmission probabilities and communication rates for all sensor nodes. (3) **Enhanced detection performance**. We show that the proposed design approach has a significant improvement in the detection performance over the classical decoupled and maximum throughput approaches. (4) **Comparison between direct transmission and in-network processing schemes**. We study the design problem when local observations are quantized, and show the conditions under which the in-network processing scheme outperforms the direct transmission scheme.

The rest of the paper is organized as follows: Section II presents the problem formulation. Section III explains the system model. Section IV presents the solution of the optimization problem to obtain the optimal TCP design. Section V presents the performance evaluation for the proposed design, in comparison to two other classical design approaches, using a numerical example. The work is concluded in Section VI.

## II. PROBLEM FORMULATION

Figure 1 illustrates the detection system architecture, where a set of  $N$  wireless sensors, and a fusion center denoted by FC, are arranged in a tree structure, and collaborate to detect the phenomenon of interest. We assume the tree structure is pre-specified, possibly based on sensor locations. Initially, the fusion center broadcasts a query message containing the location of the phenomenon to be detected, soliciting information from different sensors. Each sensor responds to its parent with the following information: (1) sensor location, (2) the average signal to noise ratio of the measured phenomenon at the sensor location, and (3) the energy the sensor allocates to the detection process .

Two approaches are possible to calculate the optimal transmission control policy. The global approach, where the fusion

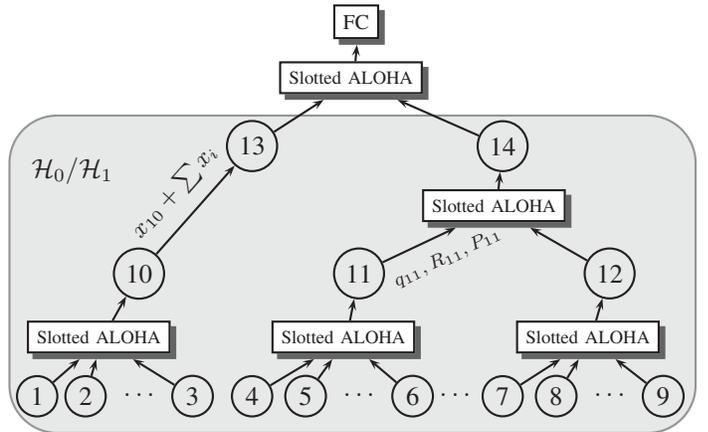


Figure 1. Detection architecture for tree-topology WSN.

center receives the information from all sensors (through their respective parents), calculates the optimal transmission control policy for each sensor by solving a constrained nonlinear optimization problem, and transmits the values of the TCP variables back to the relevant sensors. This global approach may not be feasible in large sensor networks as it is not scalable with the network size. In addition, the design parameters have to be propagated back from the fusion center to all network nodes. A more practical approach is the local approach, where each parent node solves a smaller local optimization problem to specify the *locally* optimal TCP variables for its children.

Some sensors may not contribute to the detection process, due to either low quality of information, low channel quality, or not enough energy to transmit to the parent node (e.g. not enough battery power or long distance to the parent node combined with bad channel quality). The fusion center (global approach) or the parent node (local approach) transmits the TCP variables only to the sensors which are specified by the optimization algorithm to be reliable to contribute to the detection task. The resulting values of the TCP variables remain valid for the given location as long as the quality measures for each sensor did not change from the last execution of the optimization algorithm.

After each sensor receives the optimal values of the TCP variables, the detection process proceeds as follows: The fusion center broadcasts a message to initiate a detection cycle at the local wireless sensors. Each local sensor samples the environment by collecting a number of observations, and then forms a data packet and communicates its message to the parent node over a shared wireless link using the slotted ALOHA multiaccess control scheme. Parent nodes relay the information of the child nodes, in addition to their own information, through the tree network until reaching the fusion center. Finally, the fusion center makes a final decision after a fixed amount of time representing the maximum allowed delay for detection.

In this paper, we consider the two transmission schemes. (1) *Direct transmission*, where each sensor transmits its raw observations without quantization to the fusion center. Obviously, quantization is necessary for digital communication, but the number of quantization bits is assumed large in this case so that the quantization effect is negligible. Transmission of

raw observations guarantees no loss of detection performance at the fusion center. On the down side, observations build-up and accumulate through the tree network. Therefore, the communication rate at relay nodes up in the tree hierarchy has to increase to cope with the volume of data coming from child nodes. This causes higher probability of information loss due to the high communication rate. (2) *In-network processing*, where information is compressed by calculating its Log Likelihood Ratio (LLR), then the LLR is quantized before transmission using a limited number of quantization bits. This scheme reduces the communication rate and increases the probability of successful transmission, but suffers from irrecoverable loss of information caused by the in-network processing. We assume uniform quantization to simplify the analysis, as the problem of finding the optimal quantization thresholds for detection applications have proved to be very difficult, even with small network sizes [11].

### III. SYSTEM MODEL

The detection scheme described above suggests a layered approach to system modeling, as depicted in Figure 2. The physical layer represents the wireless channel model. The Media Access Control (MAC) layer represents the slotted ALOHA protocol model. Finally, the application layer represents the sensing model, and defines the model of the observations obtained by local sensors.

#### A. Wireless Channel Model

We present a model for the wireless channel between each parent-child pair in the tree detection network. We focus on the case where the sensor nodes and the fusion center have minimal movement and the environment changes slowly. Since detection applications typically have low communication rate requirements, the coherence time of the wireless channel could be considered much larger than the transmission frame length. Accordingly, only the slow fading component of the wireless channel is considered. Figure 3 shows the fading channel model, where  $w(t)$  is an additive white Gaussian noise with power spectral density  $N_0/2$ . The term  $m(d)$  represents the mean path attenuation for a sensor node at a distance  $d$  from its parent, where the dependence on time  $t$  is dropped since slow fading is considered. We use the Hata path-loss model for the mean path attenuation, where the total dB power loss is given by [12]:

$$P_L = \underbrace{20 \log_{10} \left( \frac{4\pi d_0}{\lambda} \right) + 10\rho_c \log_{10}(d/d_0)}_{\mu_c} + X_{\sigma_c} \quad (1)$$

where  $d_0$  is a reference distance corresponding to a point located in the far field of the transmit antenna,  $\lambda$  is the

Layer	Model
App.	Sensing model
MAC	Slotted ALOHA
Physical	Fading wireless channel

Figure 2. A layered approach to detection system modeling.

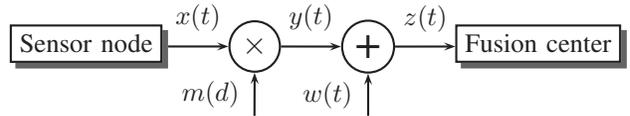


Figure 3. Block diagram for the wireless communication channel.

wavelength of the propagating signal,  $\rho_c$  is the path loss exponent,  $d$  is the distance between the transmitting and receiving antennas (i.e. child and parent nodes), and  $X_{\sigma_c}$  is a zero-mean Gaussian random variable with variance  $\sigma_c^2$ . The power loss (in dB) is therefore a Gaussian random variable with mean  $\mu_c$  and variance  $\sigma_c^2$ , i.e.  $P_L \sim \mathcal{N}(\mu_c, \sigma_c^2)$ .

The wireless channel represents an unreliable bit pipe for the data link layer, with instantaneous Shannon capacity  $C = W \log_2 \left( 1 + \frac{P_r}{N_0 W} \right)$  bps, where  $W$  is the channel bandwidth and  $P_r$  is the signal power received by the parent node. Using Shannon coding theorem, the data link layer could achieve arbitrary communication rates  $R$  up to the channel capacity using appropriate coding schemes. Given the state of the art coding schemes that approach the Shannon capacity, we can assume that the fusion center can perform error-free decoding for any transmission with bit rate  $R < C$ . Therefore, the channel is considered “ON” when  $R < C$  and “OFF” otherwise, giving rise to the two-state channel model akin to the one presented in [5]. Noting that  $P_r = P_t 10^{-P_L/10}$ , where  $P_t$  is the average signal power transmitted by the local sensor, and using the result that  $P_L \sim \mathcal{N}(\mu_c, \sigma_c^2)$ , we get the probability for the channel being “ON” during a transmission:

$$\lambda_c = \Phi \left[ \frac{1}{\sigma_c} \left( 10 \log \frac{P_t}{N_0 W (2^{\frac{R}{W}} - 1)} - \mu_c \right) \right] \quad (2)$$

where  $\Phi(\cdot)$  is the cumulative distribution function for the standard normal PDF. We note that the Channel State Information (CSI) relevant to our model is represented by the statistics  $\sigma_c, \mu_c$ , and  $N_0$ .

#### B. Media Access Control Protocol Model

We assume a slotted ALOHA multi-access communication protocol between each parent node and its child nodes, where each packet requires one time slot for the transmission, all time slots have the same length, and all transmitters are synchronized. We consider a simplified version of the MAC protocol, where collision detection, handshaking, as well as other protocol specifics are ignored to simplify the analysis. Furthermore, we assume that the sub-trees composed of each parent and its immediate children do not interfere with each other. This could be achieved in practice by using different wireless channels for transmission, or it may be as a result of the physical separation between sub-trees such that sub-tree transmissions get attenuated before interfering with other transmissions.

The detection cycle, demonstrated in Figure 4, has length  $\tau$ , which defines the *delay for detection*. The detection cycle is divided into a number of transmission slots  $L_i$ , for nodes at the same depth  $i$  of the tree, and sharing a common parent. The relationship between the number of slots for consecutive

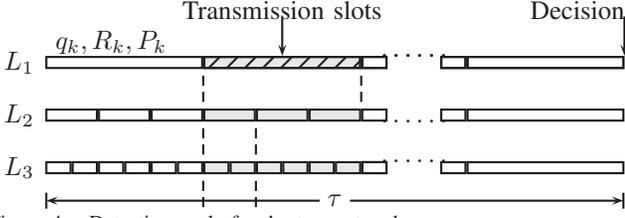


Figure 4. Detection cycle for the tree network.

depths is given by  $L_{i+1} = m_i L_i$ , where  $m_i$  is a positive integer. In the following discussion, we designate the set of all child nodes for sensor  $k$  by  $\mathcal{C}_k$ , and the set of all siblings (excluding sensor  $k$ ) by  $\mathcal{B}_k$ .

**Direct transmission.** At the beginning of every time slot, each local sensor  $k$  collects a number of observations  $n_k$  and forms an information packet for transmission over the wireless channel. The sensor then attempts to transmit to its parent with probability  $q_k$ , transmission power  $P_k$ , and communication rate  $R_k$ . The sensor attempts transmission at each time slot, despite the status of its previous transmission attempts. The final decision is taken at the fusion center using the information received during the detection cycle. The process repeats for every detection request initiated by the fusion center.

The communication rate for sensor  $k$  at tree depth  $i$  could be expressed with the aid of Figure 5 as follows:

$$R_k = \frac{bL_i n_k}{\tau} + \frac{1}{m_i} \sum_{v \in \mathcal{C}_k} Z_v R_v, \quad \sum_{v \in \mathcal{C}_k} Z_v = m_i \quad (3)$$

where  $b$  is the number of encoding bits for each observation, which is fixed, and  $Z_v$  is the number of times the child sensor  $v$  successfully transmitted during the  $m_i$  time slots. The first term in (3) represents the information collected by the sensor node, and vanishes if the node functions as a relay node for its child nodes. The second term represents the information received from the child nodes and vanishes for leaf nodes. We note here that the design variable is  $n_k$ , the number of observations collected at each time slot by sensor  $k$ .

We note that the communication rate of intermediate nodes is a random variable, being dependent on the information received from its child nodes. Accurate formulation for this problem is hard. Therefore, to keep the analysis tractable, we resort to a suboptimal solution, where the communication rate for each node is represented by its expected value:

$$\bar{R}_k = \frac{bL_i n_k}{\tau} + \sum_{v \in \mathcal{C}_k} \lambda_v \bar{R}_v \quad (4)$$

**In-Network processing.** After each sensor  $k$  collects its  $n_k$  observations in slot  $i$ , it calculates its Log Likelihood Ratio (LLR):

$$z_k = \frac{\mu^k}{\sigma_s^{k^2}} \sum_{j=1}^{n_k} x[j, i] \quad (5)$$

where  $x$  is a Gaussian random variable (as explained in Section III-D). There is no loss of optimality in this process, since the LLR is optimal at the fusion center as observations are

independent across sensors [13]. The LLR is then quantized using  $b_k$  bits, to obtain the discrete random variable  $y_k$ :

$$y_k = Q(z_k; b_k) \quad (6)$$

This quantized version is transmitted to the parent node. Each sensor node forwards the quantized LLR of its descendants without further quantization, in addition to its own quantized LLR, to the next parent node. The process repeats until all observations arrive at the fusion center. Similar to (4), the communication rate for each sensor is given by:

$$\bar{R}_k = \frac{b_k L_i}{\tau} + \sum_{v \in \mathcal{C}_k} \lambda_v \bar{R}_v \quad (7)$$

We note here that the number of quantization bits is the design variable, and it may vary for each sensor. The decision on how many quantization bits will be used is dependent on the sensor quality measures. A large number of quantization bits reduces the loss in the signal to noise ratio but increases the probability of packet loss.

Next, we calculate the overall probability of a successful packet transmission, including the wireless channel effect. At any given time slot, the probability of a single packet transmission by sensor  $k$  is given by  $q_k \prod_{v \in \mathcal{B}_k} (1 - q_v)$ . Further, this packet will be successfully received by the parent node if the state of the physical channel between the child node  $k$  and the parent node is ‘‘ON’’ during this time slot. Therefore, using (2), the total probability of a successful packet transmission by sensor  $k$  is given by:

$$\lambda_k = q_k \left[ \prod_{v \in \mathcal{B}_k} (1 - q_v) \right] \Phi \left[ \frac{10}{\sigma_c^k} \log \frac{P_t^k}{N_0 W (2^{\frac{R_k}{W}} - 1)} - \frac{\mu_c^k}{\sigma_c^k} \right] \quad (8)$$

### C. Energy Model

To formulate the energy model for each sensor, we first introduce the definition for the network lifetime. The network lifetime  $\mathcal{L}$  could be defined as the average time span from the deployment to the instant when the network can no longer perform the task [9]. The network lifetime could be expressed as:

$$\mathcal{L} = \frac{\mathcal{E}^0 - \mathcal{E}^w}{f_r \mathcal{E}^r} \quad (9)$$

where  $\mathcal{E}^0 = \sum_{i=1}^N e_i^0$  is the total initial energy in all sensors at the time of deployment,  $\mathcal{E}^w = \sum_{i=1}^N e_i^w$  is the total energy remaining in sensor nodes when the network cannot perform the assigned task,  $f_r$  is the average sensor reporting rate defined here as the number of detection cycles per unit time,

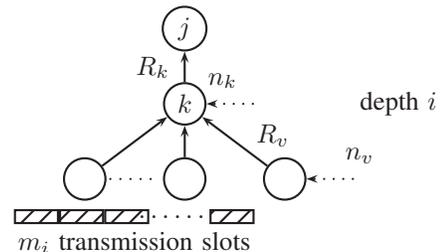


Figure 5. Communication rate calculation for node  $k$  at tree depth  $i$ .

and  $\mathcal{E}^r = \sum_{i=1}^N e_i^r$  is the expected energy consumed by all sensors in one detection cycle. The total energy remaining is defined for our detection problem as the energy required to achieve a minimum pre-specified value for the detection performance measure.

In this work, we resort to a simple energy formulation. First, we assume that  $e_i^w$  is the energy remaining in the sensor battery when the sensor is not capable of operating its electronic circuits for computations and communication, which is fixed and known for each sensor. Second, we assume that the reporting energy for each sensor  $e_i^r$  is a fixed percentage of its net useful energy at the time of sensor deployment. Using these two assumptions, we get the following expression for the energy consumed by each sensor  $k$  in one detection cycle:

$$e_k^r = \frac{e_k^0 - e_k^w}{f_r \mathcal{L}} \quad (10)$$

which could be calculated for any desired network lifetime  $\mathcal{L}$ . The total energy consumed by each sensor is divided between transmission and reception (except for leaf nodes). By assuming that the energy consumed in the reception process is proportional to the detection cycle lifetime with proportionality constant  $\alpha$ , and by noting that the expected number of transmissions by sensor  $k$  during a detection cycle is  $L_i q_k$ , we get:

$$P_t^k = \frac{(e_k^r/\tau) - \alpha}{q_k} = \frac{1}{q_k}(p_k - \alpha) \quad (11)$$

where  $p_k$  is the average transmission power over one detection cycle, which summarizes the Residual Energy Information (REI) for each sensor. Using (11) in (8), we get:

$$\lambda_k = q_k \left[ \prod_{v \in \mathcal{B}_k} (1 - q_v) \right] \Phi \left[ a_k - \left( \frac{10}{\sigma_c^k} \right) \log q_k \left( 2^{\frac{R_k}{W}} - 1 \right) \right] \quad (12)$$

where  $a_k = \frac{1}{\sigma_c^k} \left( 10 \log \frac{p_k - \alpha}{N_0 W} - \mu_c^k \right)$ . We note that  $\alpha < p_k = e_k^r/\tau$  for the sensor to be able to transmit the information. In addition,  $\alpha = 0$  for leaf nodes.

This energy formulation simplifies the analysis, as the reporting energy  $e^r$  for each sensor is preallocated. In general, however, we can include the energy allocation problem in our formulation, i.e. finding optimal  $e^r$  values for all sensors that maximize the detection performance while guaranteeing a minimum network lifetime.

#### D. Sensing Model

We consider a detection application where a set of sensors are randomly placed in a surveillance area to detect the presence of an object. Sensors have fixed positions, which could be estimated using different localization algorithms. The surveillance area is divided into a number of resolution cells that are probed by local sensors upon receiving a command from the fusion center. We focus our work on detection using signal amplitude measurements. Therefore, when there is an object at a specific resolution cell, the observation at sensor

$k$ , located at  $d_k$  distance from the object, could be expressed as:

$$x_k = \frac{\epsilon}{d_k^{\eta/2}} + w_k \quad (13)$$

where  $\epsilon$  is the amplitude of the emitted signal at the object,  $\eta$  is a known attenuation coefficient, typically between 2 and 4, and  $w_k$  is an additive white Gaussian noise with zero mean and variance  $\sigma_s^{k^2}$ .

The detection problem could be defined as the following binary hypothesis testing problem, for each time slot  $i$ :

$$\begin{aligned} \mathcal{H}_0 : x_k[j, i] &= w_k[j, i] & j = 1, 2, \dots, n_k \\ \mathcal{H}_1 : x_k[j, i] &= \mu^k + w_k[j, i] & j = 1, 2, \dots, n_k \end{aligned} \quad (14)$$

where  $\mu^k = \epsilon/d_k^{\eta/2}$ , and  $n_k$  is the number of observations obtained by sensor  $k$  at each time slot. We note that noise samples are independent across sensors, i.e., the observations at local sensors are independent across time and space, but not necessarily identically distributed since some sensors may be closer to the measured phenomenon, and noise variances are assumed unequal.

Based on the given sensing model, we next derive the objective function for the two transmission schemes.

**Direct transmission.** We present the following two propositions, without proof, due to space limitations. The detailed proofs can be found in [14].

*Proposition 1:* The optimal test statistic at the fusion center for the slotted ALOHA tree network with depth  $l$  and direct transmission scheme is given by (15).

The expression in (15) is simply a weighted sum of the observations received at the fusion center. The complexity of the equation comes from the fact that successful reception of the observations of child nodes at the fusion center depends on the success of the transmission of all parent nodes up to the fusion center.

We adopt the deflection coefficient as a detection performance measure, defined as [15]:

$$d^2 = \frac{(E[V; \mathcal{H}_1] - E[V; \mathcal{H}_0])^2}{\text{var}[V; \mathcal{H}_0]} \quad (18)$$

which provides more tractable results in our study.

*Proposition 2:* The deflection coefficient for the detector in (15) is given by (16), where  $c_v = (\mu^v/\sigma_s^v)^2$ .

We note that the quantity  $n_v c_v$  is a measure of the Quality of Information (QoI) for each sensor. Using (4) in (16), we obtain our objective function in (17).

**In-Network Processing.** We first choose the test statistic in (22) to be implemented at the fusion center. This test statistic is sub-optimal. To obtain the optimal one, we need to take the LLR for the discrete random variables  $Y_i$  in (6) at the fusion center. Unfortunately, this problem does not have a closed form solution, and the detector performance is usually approximated using different statistical techniques [16]. We resort to the suboptimal statistic in (22), as it is similar to the one in (15) for the direct observation system, which facilitates the performance comparison. Now, to find the deflection coefficient for the statistic in (22), we need to calculate the expectation of  $V$  under both  $\mathcal{H}_0$  and  $\mathcal{H}_1$ , in

$$V = \sum_{i_1=1}^{L_1} \sum_{v_1 \in \mathcal{C}_f} \sum_{j_1=1}^{n_{v_1}} r_{v_1}[i_1] \left[ \left( \frac{\mu^{v_1}}{\sigma_s^{v_1/2}} \right) x_{v_1}[j_1, i_1] + \dots + \sum_{i_l=1}^{m_{l-1}} \sum_{v_l \in \mathcal{C}_{v_{l-1}}} \sum_{j_l=1}^{n_{v_l}} r_{v_l}[i_1 \dots i_l] \left( \frac{\mu^{v_l}}{\sigma_s^{v_l/2}} \right) x_{v_l}[j_l, i_1 i_2 \dots i_l] \right] \quad (15)$$

$$D^2 = L_1 \sum_{v_1 \in \mathcal{C}_f} \lambda_{v_1} \left[ n_{v_1} c_{v_1} + m_1 \sum_{v_2 \in \mathcal{C}_{v_1}} \lambda_{v_2} \left[ n_{v_2} c_{v_2} + \dots + m_{d-1} \sum_{v_l \in \mathcal{C}_{v_{l-1}}} \lambda_{v_l} n_{v_l} c_{v_l} \right] \dots \right] \quad (16)$$

$$D^2 = \frac{\tau}{b} \sum_{v_1 \in \mathcal{C}_f} \lambda_{v_1} \left[ \bar{R}_{v_1} c_{v_1} + \sum_{v_2 \in \mathcal{C}_{v_1}} \lambda_{v_2} \left[ \bar{R}_{v_2} (c_{v_2} - c_{v_1}) + \sum_{v_3 \in \mathcal{C}_{v_2}} \lambda_{v_3} \left[ \bar{R}_{v_3} (c_{v_3} - c_{v_2}) + \dots + \sum_{v_l \in \mathcal{C}_{v_{l-1}}} \lambda_{v_l} \bar{R}_{v_l} (c_{v_l} - c_{v_{l-1}}) \right] \dots \right] \right] \quad (17)$$

addition to its variance under  $\mathcal{H}_0$ . We first need to define the quantization function in (6). We adopt the following quantizer:

$$Q(z) = \Delta \left( \left\lfloor \frac{z}{\Delta} \right\rfloor + \frac{1}{2} \right) \quad (19)$$

where  $\Delta$  is the quantizer step size. We have the following proposition.

*Proposition 3:* The deflection coefficient of the test statistic in (22), with the quantizer in (19), is given by (23), where:

$$\delta = \frac{\Delta}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin \left[ \frac{2\pi k n}{\Delta} \left( \frac{\mu}{\sigma_s} \right)^2 \right] e^{-2 \left( \frac{\pi k}{\Delta} \right)^2 n \left( \frac{\mu}{\sigma_s} \right)^2} \quad (20)$$

and  $\delta'$  is given by (24).

The expression for the deflection coefficient in (23) is not amenable to optimization. However, we note that both the mean and variance degrade exponentially with the quantizer step size  $\Delta$ . Since  $\Delta$  is inversely proportional to the number of quantization bits,  $b_k$ , we can approximate the signal to noise ratio for each sensor  $k$  after quantization by:

$$S = n_k \left( \frac{\mu_k}{\sigma_k^2} \right)^2 (1 - 2^{-\beta b_k}) \quad (21)$$

where  $\beta$  specifies the decay rate, and depends on the range of the quantizer as well as the quantizer design. Now, we use the degraded SNR in (21) to define our approximate deflection coefficient as in (25), where  $c'_v = (\mu^v / \sigma_s^v)^2 (1 - 2^{-\beta b_v})$

#### IV. TCP DESIGN FOR OPTIMAL DETECTION

**Direct transmission.** The optimization problem could be summarized as follows:

$$\begin{aligned} \max_{\mathbf{q}, \bar{\mathbf{R}}} \frac{\tau}{b} \sum_{v_1 \in \mathcal{C}_f} \lambda_{v_1} \left[ \bar{R}_{v_1} c_{v_1} + \sum_{v_2 \in \mathcal{C}_{v_1}} \lambda_{v_2} \left[ \bar{R}_{v_2} (c_{v_2} - c_{v_1}) + \dots + \sum_{v_l \in \mathcal{C}_{v_{l-1}}} \lambda_{v_l} \bar{R}_{v_l} (c_{v_l} - c_{v_{l-1}}) \right] \dots \right] \\ \text{s.t. } 0 \leq q_i \leq 1, \bar{R}_i \geq \sum_{v \in \mathcal{C}_i} \lambda_v \bar{R}_v \quad i = 1 : N \end{aligned} \quad (27)$$

where:

$$\begin{aligned} \lambda_v = q_v \left[ \prod_{k \in \mathcal{B}_v} (1 - q_k) \right] \Phi \left[ a_v - \left( \frac{10}{\sigma_c^v} \right) \log q_v \left( 2^{\frac{\bar{R}_v}{W}} - 1 \right) \right] \\ a_v = \frac{1}{\sigma_c^v} \left( 10 \log \frac{p_v - \alpha}{N_0 W} - \mu_c^v \right), c_v = \left( \frac{\mu^v}{\sigma_s^v} \right)^2 \end{aligned} \quad (28)$$

The last constraint guarantees that intermediate nodes can at least relay the observations of their children nodes. This constraint reduces to  $\bar{R}_i \geq 0$  for leaf nodes. Although this problem could be solved by existing algorithms (e.g. interior point methods) for a local maximum, we note that the objective function in (17) gets more complicated as the tree depth increases. Adding the fact that all design variables have to be propagated back to tree nodes, a more practical approach is clearly needed. If we look at the objective function expression in (17), we note that it reflects the tree hierarchy, i.e. the last term in the expression represents the contribution of the leaf nodes, preceded by the contribution of the parents of the leaf nodes, and so on, until reaching the sensor nodes at the top level of the tree (depth=1). This could be shown by expressing the objective function using the following recursive equation:

$$D^2 = \frac{\tau}{b} J_{FC}, J_k = \sum_{v \in \mathcal{C}_k} \lambda_v [\bar{R}_v (c_v - c_k) + J_v] \quad (29)$$

where  $J_v = 0$  for leaf nodes and  $c_k = 0$  for the fusion center node. This structure of the objective function suggests a local optimization approach for the problem, where we start by optimizing  $J_v$  for sensors at depth  $l-1$  and continue the local optimization recursively using (29), until reaching the fusion center. This approach is practical since the solution of each local optimization problem could be carried out locally at each parent node. The solution approach is illustrated in Figure 6. By substituting (12) in (29), we can express the local optimization problem at parent node  $k$  as follows:

$$\begin{aligned} \max_{\mathbf{q}, \bar{\mathbf{R}}} \sum_{v \in \mathcal{C}_k} q_v \left[ \prod_{i \in \mathcal{B}_v} (1 - q_i) \right] [\bar{R}_v (c_v - c_k) + J_v] \times \\ \Phi \left[ a_v - \left( \frac{10}{\sigma_c^v} \right) \log q_v \left( 2^{\frac{\bar{R}_v}{W}} - 1 \right) \right] \\ \text{s.t. } 0 \leq q_v \leq 1, \bar{R}_v \geq \sum_{u \in \mathcal{C}_v} \lambda_u \bar{R}_u = r_v \end{aligned} \quad (30)$$

We note that  $J_v$  and  $r_v$  are fixed values, obtained from solving the local optimization problems at lower levels in the hierarchy. The notation for the local optimization problem is illustrated in Figure 7.

Let the number of child nodes for sensor  $k$  be  $N_k$ , and denote the decision variables by:

$$\mathbf{x} = [q_1 \quad q_2 \quad \dots \quad q_{N_k} \quad \bar{R}_1 \quad \bar{R}_2 \quad \dots \quad \bar{R}_{N_k}] \quad (31)$$

where  $\mathbf{x} \in \mathbb{R}^{2N_k}$ , and the objective function by  $J(\mathbf{x})$ , then the optimization problem could be rewritten as:

$$\min_{\mathbf{x}} -J(\mathbf{x}) \quad \text{subject to } A\mathbf{x} \geq \mathbf{b} \quad (32)$$

$$V = \sum_{i_1=1}^{L_1} \sum_{v_1 \in \mathcal{C}_f} r_{v_1} [i_1] \left[ y_{v_1} + \sum_{i_2=1}^{m_1} \sum_{v_2 \in \mathcal{C}_{v_1}} r_{v_2} [i_1 i_2] \left[ y_{v_2} [j_2, i_1 i_2] + \dots + \sum_{i_l=1}^{m_{l-1}} \sum_{v_l \in \mathcal{C}_{v_{l-1}}} r_{v_l} [i_1 \dots i_l] y_{v_l} [j_l, i_1 i_2 \dots i_l] \right] \dots \right] \quad (22)$$

$$D^2 = \frac{\left( L_1 \sum_{v_1 \in \mathcal{C}_f} \lambda_{v_1} \left[ n_{v_1} \left( \frac{\mu_{v_1}^1}{\sigma_s^1} \right)^2 + \delta'_{v_1} + m_1 \sum_{v_2 \in \mathcal{C}_{v_1}} \lambda_{v_2} \left[ n_{v_2} \left( \frac{\mu_{v_2}^2}{\sigma_s^2} \right)^2 + \delta'_{v_2} \dots m_l \sum_{v_l \in \mathcal{C}_{v_l}} \lambda_{v_l} n_{v_l} \left( \frac{\mu_{v_l}^l}{\sigma_s^l} \right)^2 + \delta'_{v_l} \right] \dots \right)^2}{L_1 \sum_{v_1 \in \mathcal{C}_f} \lambda_{v_1} \left[ n_{v_1} \left( \frac{\mu_{v_1}^1}{\sigma_s^1} \right)^2 + \delta'_{v_1} + m_1 \sum_{v_2 \in \mathcal{C}_{v_1}} \lambda_{v_2} \left[ n_{v_2} \left( \frac{\mu_{v_2}^2}{\sigma_s^2} \right)^2 + \delta'_{v_2} + \dots m_l \sum_{v_l \in \mathcal{C}_{v_l}} \lambda_{v_l} n_{v_l} \left( \frac{\mu_{v_l}^l}{\sigma_s^l} \right)^2 + \delta'_{v_l} \right] \dots \right]} \quad (23)$$

$$\delta' = \left( \frac{\Delta}{\pi} \right)^2 \left[ \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{1}{2k_1 k_2} \left( \cos \left[ \frac{2\pi(k_1 - k_2)n}{\Delta} \left( \frac{\mu}{\sigma_s} \right)^2 \right] e^{-2 \left( \frac{\pi(k_1 - k_2)}{\Delta} \right)^2 n \left( \frac{\mu}{\sigma_s} \right)^2} - \cos \left[ \frac{2\pi(k_1 + k_2)n}{\Delta} \left( \frac{\mu}{\sigma_s} \right)^2 \right] e^{-2 \left( \frac{\pi(k_1 + k_2)}{\Delta} \right)^2 n \left( \frac{\mu}{\sigma_s} \right)^2} \right) \right] \quad (24)$$

$$D^2 = L_1 \sum_{v_1 \in \mathcal{C}_f} \lambda_{v_1} \left[ n_{v_1} c'_{v_1} + m_1 \sum_{v_2 \in \mathcal{C}_{v_1}} \lambda_{v_2} \left[ n_{v_2} c'_{v_2} + \dots + m_{d-1} \sum_{v_l \in \mathcal{C}_{v_{l-1}}} \lambda_{v_l} n_{v_l} c'_{v_l} \right] \dots \right] \quad (25)$$

$$D^2 = \frac{\tau}{b} \sum_{v_1 \in \mathcal{C}_f} \lambda_{v_1} \left[ u_{v_1} c'_{v_1} + \sum_{v_2 \in \mathcal{C}_{v_1}} \lambda_{v_2} \left[ u_{v_2} c'_{v_2} + \sum_{v_3 \in \mathcal{C}_{v_2}} \lambda_{v_3} \left[ u_{v_3} c'_{v_3} + \sum_{v_4 \in \mathcal{C}_{v_3}} \dots + \sum_{v_l \in \mathcal{C}_{v_{l-1}}} \lambda_{v_l} u_{v_l} c'_{v_l} \right] \right] \dots \right] \quad (26)$$

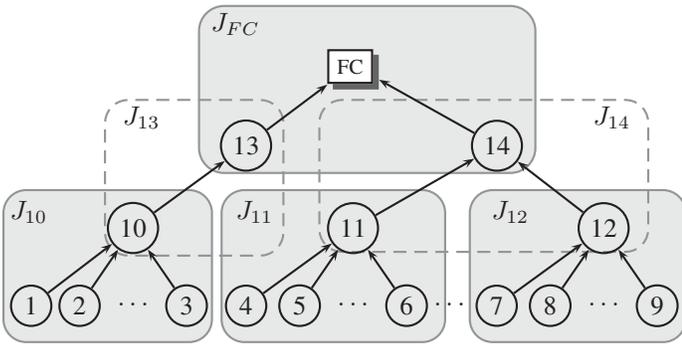


Figure 6. Hierarchical optimization for the TCP design problem.

where

$$A = \begin{bmatrix} I & -I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I \end{bmatrix}^T, \quad \mathbf{b} = [\mathbf{0} \quad -\mathbf{1} \quad \mathbf{r}]^T \quad (33)$$

$I$  is the identity matrix,  $\mathbf{0}(\mathbf{1})$  is the vector/matrix of all zeros (ones) with appropriate dimensions, and  $\mathbf{r} = [r_1 \ r_2 \ \dots \ r_{N_k}]^T$ . Although the objective function is not convex, we note that the inequality constraints are linear. Therefore, the Karush-Kuhn-Tucker (KKT) conditions are necessary conditions for a local maximizer of the objective function [17]. We present the following proposition without proof, due to space limitations. The result is derived by solving the KKT conditions.

*Proposition 4:* The maximum value of the objective function in (30) occurs either when one sensor transmits with probability one and all other sensors remain silent, or at a stationary point of the objective function, i.e. at  $\mathbf{x}^*$  where  $\nabla J(\mathbf{x}^*) = \mathbf{0}$ .

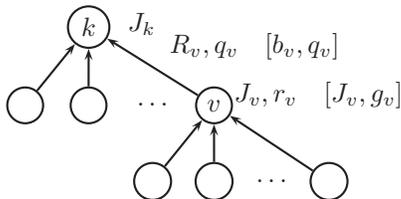


Figure 7. Notation for the local optimization problem. In-network processing parameters are shown between brackets.

Since we may have multiple stationary points in the interior of the objective function domain, the proposition does not guarantee obtaining the global maximum. However, the proposition is still useful for the following reasons: 1) it avoids the case where the optimization algorithm may terminate at the local maximum  $q_i = 1, q_j = 0$ , while a better local maximum maybe at one of the stationary points, and 2) it provides information about the choice of the initial point for the optimization algorithm, where initial points near the corner points  $q_i = 1, q_j = 0$  have to be avoided.

**In-Network Processing.** We note that the objective function in (26) has the same recursive structure as the direct transmission design. Accordingly, we adopt the same local approach presented for the direct transmission scheme to solve for the optimal design variables. The local optimization problem at parent node  $k$  could be expressed as <sup>1</sup>:

$$\begin{aligned} \max_{q, b} \sum_{v \in \mathcal{C}_k} q_v \left[ \prod_{i \in \mathcal{B}_v} (1 - q_i) \right] \left[ u_v \left( \frac{\mu^v}{\sigma_s^v} \right)^2 (1 - 2^{-\beta b_v}) + J_v \right] \times \\ \Phi \left[ a_v - \left( \frac{10}{\sigma_c^v} \right) \log q_v \left( 2^{[b_v(L/\tau) + g_v]/W} - 1 \right) \right] \\ \text{s.t. } 0 \leq q_v \leq 1, b_v \in \mathbb{N} \end{aligned} \quad (34)$$

where  $u_v = \bar{R}_v - r_v$ , and  $r_v$  is the average communication rate for the child nodes  $\mathcal{C}_v$ . We note that  $J_v$  and  $g_v$  are fixed values, obtained from solving the local optimization problems at lower levels in the hierarchy. The notation for the local optimization problem with in-network processing is illustrated in Figure 7.

## V. PERFORMANCE EVALUATION

We compare our design approach to the classical decoupled and maximum throughput design approaches. The optimization for the classical designs is carried out using the same local approach pursued for the proposed design.

### A. Decoupled design

In this approach, each layer is designed separately. In the conventional slotted ALOHA, the MAC sublayer is designed

<sup>1</sup>Details are omitted due to space limitation

to minimize the probability of collision, without regard to the QoI and CSI for each node. For the sub-tree composed of node  $k$  and the set of its immediate children,  $\mathcal{C}_k$ , minimum probability of collision occurs at  $q_v = 1/N_k$ , where  $N_k = |\mathcal{C}_k|$ , and consequently  $P_t^k = (p_k - \alpha)/N_k$ . The physical layer is designed to guarantee a minimum probability of successful packet transmission,  $\lambda_v$ . Using (2), we obtain:

$$\bar{R}_i = W \log_2 \left( 1 + 10^{[0.1\sigma_c^i(a_i - \Phi^{-1}[\lambda_v]) + \log N_k]} \right) \quad (35)$$

and using (29), the deflection coefficient is given by:

$$D^2 = \frac{\tau}{b} J_{FC}$$

$$J_k = \frac{\lambda_v}{N_k} \left( 1 - \frac{1}{N_k} \right)^{N_k-1} \sum_{v \in \mathcal{C}_k} [J_v + (c_v - c_k) \bar{R}_i] \quad (36)$$

To make a fair comparison, we do not assume a pre-set value of  $\lambda_k$ . Rather, we optimize  $\lambda_k$  values to yield the maximum deflection coefficient. Therefore, the local optimization problem in (37) could be written on the form:

$$\max_{\lambda_v} \frac{\lambda_v}{N_k} \left( 1 - \frac{1}{N_k} \right)^{N_k-1} \sum_{v \in \mathcal{C}_k} [\bar{R}_v (c_v - c_k) + J_v]$$

$$\text{s.t. } \bar{R}_v \geq \sum_{u \in \mathcal{C}_v} \lambda_u \bar{R}_u = r_v \quad (37)$$

where  $\bar{R}_v$  is given by (35).

### B. Max. Throughput Design

The throughput of any relay node  $k$  is defined as:

$$T_k = \sum_{v \in \mathcal{C}_k} \lambda_v \bar{R}_v$$

$$= \sum_{v \in \mathcal{C}_k} \bar{R}_v q_v \prod_{i \in \mathcal{B}_v} (1 - q_i) \Phi \left[ a_v - \left( \frac{10}{\sigma_c^v} \right) \log q_v \left( 2 \frac{\bar{R}_v}{W} - 1 \right) \right] \quad (38)$$

The objective is to choose the design variables  $q_v$  and  $R_v$  to locally maximize the throughput. The constraint on the communication rate of node  $v$  could be expressed in terms of its throughput as  $\bar{R}_v \geq T_v$ , where  $T_v = 0$  for leaf nodes. The optimization problem could be formulated as:

$$\max_{q_v, \bar{R}_v} T_k \quad \text{s.t. } 0 \leq q_v \leq 1, \bar{R}_v \geq \sum_{u \in \mathcal{C}_v} \lambda_u \bar{R}_u \quad (39)$$

where  $\lambda_u$  and  $\bar{R}_u$  are obtained from solving the local optimization problems at the lower level for each node  $v$ , as indicated in Figure 7. The optimal design variables could then be substituted back in (29) to evaluate the deflection coefficient.

### C. Numerical Example

We consider the tree network in Figure 8, with system parameters as indicated on the tree edges. We use  $W = 2 \times 10^3$  Hz,  $N_0 = 10^{-10}$  W/Hz, and  $b = 16$  bits. For each value of the delay for detection  $\tau$  and network lifetime  $\mathcal{L}$ , we solve the local optimization problems in (30), (34), (37), and (38) recursively to find the optimal TCP design variables

and the relevant deflection coefficient value for each design approach. We calculate the optimal solution in each case using the fmincon interior-point algorithm in Matlab Optimization Toolbox [18].

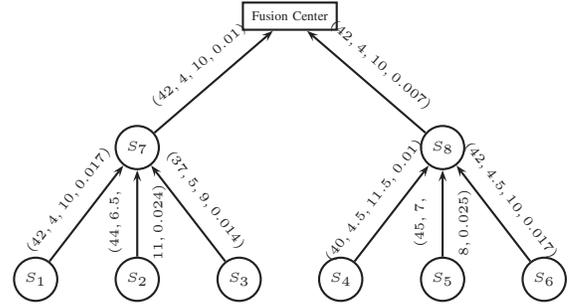


Figure 8. Tree detection network for the example problem. Labels on each edge represent  $\mu_c, \sigma_c, e$  (in mJ), and Signal to noise ratio, respectively, for each source sensor.

**Proposed design performance.** Figure 9 shows the performance surface for the slotted ALOHA tree network in Figure 8, for the proposed design approach. The surface plots the deflection coefficient for different delay and network lifetime values. For a fixed network lifetime, the deflection coefficient increases with the delay for detection, as more observations are expected at the fusion center. For a fixed delay for detection, the deflection coefficient decreases with network lifetime, as the energy budget allocated for each detection cycle decreases to prolong the network lifetime. Decreasing the energy budget reduces the probability of successful packet transmission, hence causing less observations at the fusion center.

Figure 10 shows a contour plot for the deflection coefficient, where each curve corresponds to the set of pair values (Delay for detection, network lifetime) that gives rise to the indicated value of the deflection coefficient. To keep the deflection coefficient constant while increasing the network lifetime, the delay for detection has to increase also, so that more observations could be received in each detection cycle. This compensates for the energy decrease as a result of a prolonged network lifetime.

**Performance comparison.** We resort to one dimensional plots to compare between the different design approaches. Figure 11 shows the deflection coefficient versus the delay for detection, where the network lifetime is set to 250 days. The decoupled design approach exhibits the worst performance,

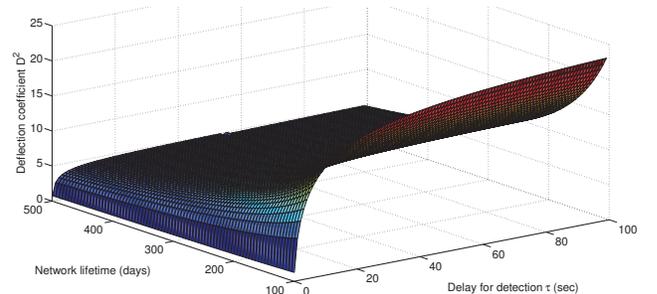


Figure 9. Deflection coefficient as it varies with the network lifetime and delay for detection.



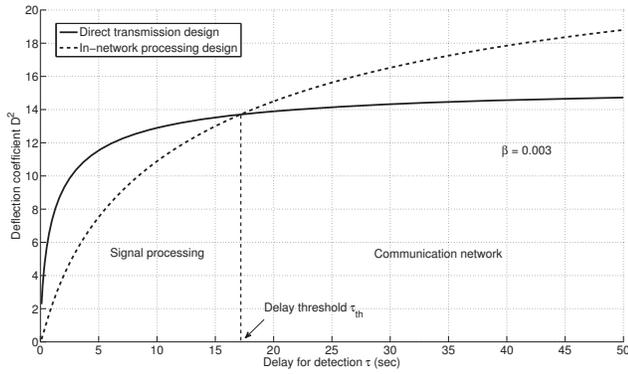


Figure 13. Deflection coefficient for direct transmission and in-network processing designs.

to noise ratio for each sensor and the energy allocated for the detection process. Figure 14 shows the variation in the delay threshold with the quantizer design parameter  $\beta$ . As  $\beta$  increases, the exponential decay rate for quantization effect is much faster, hence the threshold is lower.

## VI. CONCLUSIONS

In this paper, we used a model-based approach to design a tree-structured, slotted ALOHA sensor network, for detection applications. We developed an integrated model for the detection system and integrated the QoI, REI, and CSI quality measures into the design process. We designed the communication rate and transmission probability for each tree node. The proposed model-based approach shows significant performance gain over the classical decoupled and maximum throughput approaches commonly adopted to design sensor networks. This performance enhancement comes with no additional complexity since the optimization problem is very similar to the max. throughput design case.

For applications with stringent delay requirements, we show that system design with direct transmission of sensor observations results in better performance since the channel impairment is unlikely to play a major role. If the application can tolerate longer delays, then the design with in-network processing results in better detection performance, as the communication network becomes a dominant factor in determining

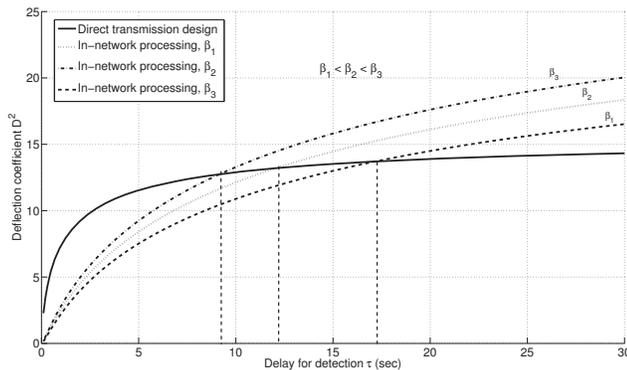


Figure 14. Variation of detection threshold with quantizer parameter  $\beta$ .

the system performance.

We assumed that the energy is preallocated to each sensor based on its energy reserve. Optimal energy allocation to maximize the detection performance is one possible extension to the presented work. In this case, care should be taken that the energy of relay nodes is not depleted before its descendants. Another possible extension is the design of TDMA tree networks, where all transmissions are pre-scheduled, and the performance comparison to the ALOHA network presented in this paper.

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