On hybrid control of complex systems: a survey

Sur la commande hybride des systèmes complexes : état de l'art

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ABSTRACT. In this paper a brief overview of hybrid control systems is given and an introduction to several approaches in hybrid systems research with an extended list of references is presented. Emphasis is put on Petri net approaches to hybrid control.

RESUME. L'objectif de cet article est de présenter un état de l'art des travaux relatifs au contrôle-commande des systèmes hybrides. Plusieurs approches de recherche sur les systèmes hybrides sont abordées et une liste étendue de références est proposée. Les approches à base de réseaux de Petri sont plus particulièrement abordées.

KEY WORDS: Hybrid systems, modeling, verification, synthesis, Petri nets

MOTS-CLES : Systèmes hybrides, modélisation, vérification, synthèse, réseaux de Petri

1. Introduction

The hybrid systems considered in this paper are dynamical systems, where the behavior of interest is determined by interacting continuous and discrete dynamics. These systems typically contain variables or signals that take values from a continuous set (e.g. the set of real numbers) and also variables that take values from a discrete, typically finite set (e.g. the set of symbols {a, b, c}). These continuous or discrete-valued variables or signals depend on independent variables such as time, which may also be continuous or discrete; some of the variables may also be discrete event driven in an asynchronous manner.

There are several reasons for using hybrid models to represent the dynamic behavior of interest. Reducing complexity was and still is an important reason for dealing with hybrid systems; this is accomplished by incorporating models of dynamic processes having different levels of abstraction. For example a thermostat typically sees a very simple, but adequate for the task in hand, model of the complex heat flow dynamics; for another example, in order to avoid dealing directly with a set of nonlinear equations one may choose to work with sets of simpler equations (e.g. linear), and switch among these simpler models. The advent of digital machines has made hybrid systems very common indeed. Whenever a digital device interacts with the continuous world, the behavior involves hybrid phenomena that need to be analyzed and understood.

Hybrid control systems typically arise from computer aided control of continuous processes, manufacturing and communication networks for example; also from the hierarchical organization of complex control systems to manage complexity. Note that in the later case, higher levels in the hierarchy require less detailed models (discrete abstractions) of the functioning of the lower levels, necessitating the interaction of discrete and continuous components. The study of hybrid control systems is essential in designing sequential supervisory controllers for continuous systems, and it is central in designing intelligent control systems with a high degree of autonomy. The investigation of hybrid systems is creating a new and fascinating discipline bridging control engineering, mathematics and computer science.

Even though the study of hybrid systems as an individual area of control is relatively new, (see for example [QUE 94], [ENG 97]), many of the kinds of systems which fall under this category have been studied before. Important fields of study ancestral to hybrid control are bang-bang control, sliding mode control, variable structure control, and digital control. Approaches to hybrid systems appeared in [WIT 66], [JOH 81]. More recently, the study of hybrid systems in [GOL 89], [PEL 89], [BEN 90], [STI 91] helped raise the awareness of the research community of the importance of this field of study.

In the following, after an introduction to major approaches in hybrid systems research, specific contributions are discussed, with modeling issues and approaches being addressed first. The last part of the paper reviews a number of approaches that involve the use of Petri nets for modeling, analysis and synthesis of hybrid systems. Note that in this paper we have put emphasis on Petri net approaches to hybrid control systems and to recent results presented at the ADPM'98 Conference and we refer the reader to the references for further details. It should be noted that due to paper length limitations the list and descriptions of approaches are brief and not complete by far. It is hoped that this paper will provide a useful and representative description of main approaches to hybrid control together with references, and as such it will be a useful resource to students, researchers and engineers.

2. Main approaches in hybrid systems

A look at the literature shows that there are many approaches to modeling, analysis and synthesis of hybrid systems. They can be characterized and described along several dimensions. In broad terms, approaches differ with respect to the emphasis on or the complexity of the continuous and discrete dynamics, and on whether they emphasize analysis and synthesis results or analysis only or simulation only. On one end of the spectrum there are approaches to hybrid systems that represent extensions of system theoretic ideas for systems (with continuous-valued variables and continuous time) that are described by ordinary differential equations to include discrete time and variables that exhibit jumps, or extend results to switching systems. Typically these approaches are able to deal with complex continuous dynamics and emphasize stability results. On the other end of the spectrum there are approaches to hybrid systems that are embedded in computer science models and methods, that represent extensions of verification methodologies from discrete systems to hybrid systems. Typically these approaches are able to deal with complex discrete dynamics described by finite automata and emphasize analysis results (verification) and simulation methodologies. There are additional methodologies spanning the rest of the spectrum that combine concepts from continuous control systems described by linear and nonlinear differential/difference equations, and from supervisory control of discrete event systems that are described by finite automata and Petri nets to derive, with varying success, analysis and synthesis results.

There are analogies between certain current approaches to hybrid control and digital control systems methodologies. Specifically, in digital control one could carry the control design in the continuous time domain, then approximate or emulate the controller by a discrete controller and implement it using an interface consisting of a sampler and a hold device (A/D and D/A respectively). Alternatively, one could obtain first a discrete model of the plant taken together with the interface and then carry the controller design in the discrete domain. In hybrid systems, in a manner analogous to the latter case, one may obtain a discrete event model of the plant together with the interface using automata or Petri nets; the controller is then designed using DES supervisor methodologies. Approaches analogous to the former also exist.

Further information on hybrid systems may be found in references [GRO 93], [PNU 95], [ANT 95], [ALU 96], [ANT 97], [ANT 98a, b, c], [MAL 97], [MOR 97], [HEN 98a], [ZAY 98].

2.1. *Modelling Approaches*

Several different mathematical paradigms have been used for modeling hybrid systems revealing the diversity of the researches. Tavernini [TAV 87] used differential automata, Nerode and Kohn [NER 93a] took an automata theoretic approach to systems composed of interacting Ordinary Differential Equations (ODEs) and finite automata, Antsaklis et al. [ANT 93], [STI 96a] took a discrete event dynamical systems approach; Brockett [BRO 93] combined ODEs and discrete phenomena to describe motion systems, Back et al. [BAC 93] provided a framework suitable for numerical simulation. Alur et al. [ALU 93] used hybrid automata, an extension of timed automata [ALU 94]; Chaochen et al. [CHA 93] used Duration Calculus for hybrid real-time systems, and Benveniste [BEN 98] proposed a behavioral framework of hybrid systems modeling with emphasis on compositionality and the use of multiform time. Buisson and Cormerais [BUI 98] used bond graphs to model switched continuous subsystems of linear hybrid systems; Carpanzano and Ferrarini [CAR 98b], and Soriano [SOR 98] studied the use of object-oriented paradigms for the design of hybrid systems; Savignac and Bergeon [SAV 98] used semantic networks and task trees to represent the structural knowledge of supervision systems for hybrid controllers.

Traditional control theory is based on the continuity of the maps from the output measurements to control inputs, so that small changes in the input cause small changes in the output. In [NER 93a] these topological issues are studied for hybrid systems and small topologies are introduced for the design of the analog-to-digital map. In [BRA 94], [BRA 98a] a unified hybrid systems model is introduced, which captures many discrete phenomena arising in hybrid systems. These phenomena include autonomous switchings, which refer to the discontinuous changes of the vector field describing the dynamics of the system when the state hits certain boundaries, and controlled switchings when the vector field changes abruptly in response to a control command. In the case when the state jumps discontinuously on hitting prescribed regions or in response to a control command, we have autonomous or controlled jumps respectively.

2.2. *Analysis, Synthesis and Simulation*

In the following, the approaches outlined are arranged from papers that extend conventional system theoretic results and emphasize differential/difference equation approaches, to papers that incorporate automata models and verification ideas from computer science.

In the work by Ye et al. [YE 98], a model which is suitable for qualitative analysis of hybrid dynamical systems is presented. The notion of an invariant set (e.g. equilibrium) and several types of (Lyapunov-like) stability concepts for the invariant set are defined. Sufficient conditions for uniform stability, uniform asymptotic stability, exponential stability and instability are established. Necessary conditions (converse theorems) for some of the above stability types are also established. In addition, sufficient conditions for the uniform boundedness of the motions of hybrid systems (Lagrange stability) are given. Examples include sampled-data feedback control systems, systems with impulse effects, switched systems.

In [BRA 98b] some analysis tools for switched and hybrid systems are presented. In particular, multiple Lyapunov functions are used for stability analysis of switched systems and iterated function systems are used for Lagrange stability.

In [JOH 98a] a computational approach to stability analysis of nonlinear and hybrid systems is presented. The search of piecewise quadratic Lyapunov function is formulated as a convex optimization problem in terms of linear matrix inequalities (LMI).

In [PET 96] stability and robustness issues of hybrid systems based on Lyapunov theory are presented. The authors are concerned about the applicability of the results and they proposed strong conditions for stability in order to formulate the search for Lyapunov functions as an LMI problem. Nenninger and Krebs [NEN 98] present a hybrid modeling framework that allows to examine hybrid reachability and stability of hybrid equilibrium points and their domain of attraction.

Franke [FRA 98] considers hybrid systems with a background in resource to task allocation problems with competing subtasks that usually do not allow for a steady state to be taken. Inherent disturbances usually make the over-all system a non-autonomous one. Since classical notion of Lyapunov stability is not adequate for this type of problems, the author introduces a more realistic notion of stability, and proposes a 2D-approach to worst-case analysis, inspired by the two-dimensional system theory [AMA 98]. A three tank switched server arrival system, modeled as a continuous-discrete 2D-system, is used for illustration.

The paper by van de Schaft and Schumacher [SCH 98] studies the well-posedness (existence and uniqueness of solutions) of a special class of hybrid systems, which are called complementarity systems. These systems are related to the linear complementarity problem of mathematical programming. First, complementarity modeling is presented and well-posedness is defined. Sufficient conditions for the uniqueness of smooth continuations of complementarity systems of arbitrary number of discrete states are established.

The work by Kolmanovsky and McClamroch [KOL 96] presents results for the feedback stabilization of a class on nonlinear systems using hybrid feedback controllers. The systems studied, which can be viewed as a cascade of an linear time invariant and a nonlinear system, are general enough and can arise in many real control problems. The controller is constructed to induce a two time scale behavior, slow and fast dynamics, and essentially feedback linearizes the original nonlinear system at the two different time scales, an approach which is intuitive and natural in many control applications.

Guckenheimer [GUC 95], and McClamroch et al. [MCC 97] take a nonlinear control perspective, where switching is used to expand the domain of attraction of a nonlinear control system. The nonlinear control system admits a family of equilibria corresponding to constant control inputs. The idea is to switch at discrete time instants from a control input to another in a way that the system gradually progresses from one equilibrium to another towards the final equilibrium.

In [STI 96a] a model is introduced that describes the continuous plant and discrete event controller along with an interface. A DES automaton description is employed to describe the plant together with the interface and it is used to analyze the hybrid control system. The notion of determinism is introduced and controllability is defined for hybrid control systems and it is used to obtain a controller design method. An alternative procedure for the design of the discrete event controller of this model is proposed by Oltean [OLT 98]. Using the same model, a method for control design, based on the natural invariants of the continuous part, has been developed in [STI 96b].

Tittus and Egardt [TIT 98] study control design for a class of hybrid systems with continuous dynamics described by pure integrators. Although this class of hybrid systems is very limited, these models are very important for control of batch processes. The paper introduces a notion of controllability and proposes a controllability analysis that is formulated as a backward-reachability problem. The analysis is based on a hybrid automaton model, and the framework consists of a hybrid plant and a hybrid controller that interact in a feedback fashion.

Hybrid systems consisting of switched integrators are also studied by Moor [MOO 98]. In this contribution, a finite automaton, not necessarily deterministic, is proposed as a condensed model, suitable for locating cyclic trajectories. A control law is set up in order to force a given cyclic trajectory to become globally attractive with respect to the events of the discrete interface of the switching system.

A DES approach is followed also by Raisch and O'Young, [RAI 98a]. The paper addresses the problem where a continuous plant is to be controlled via symbolic feedback. The hybrid problem is first translated into a purely discrete problem by approximating the continuous plant model by a nondeterministic finite state machine. By taking into account past measurements and control symbols, approximation accuracy can be improved and adjusted to the specification requirements. Supervisory control theory for discrete event systems is then applied to find the optimal controller which enforces the specifications. In [RAI 98b], an input/output point of view is adopted to further improve the approximation accuracy.

Cury and coworkers [CUR 98] study supervisory control for a class of continuous-time hybrid systems. The supervisor is allowed to switch the discrete-valued input signal when threshold events are observed. The objective is to synthesize a nonblocking supervisor such that the set of possible sequences of control and threshold event pairs for the closed-loop system lies between given upper and lower bounds in the sense of set containment. It is shown how this problem can be converted into a supervisor synthesis problem for a standard controlled DES. A finite representation may not exist for the exact DES model of the hybrid system, however. To circumvent this difficulty, an algorithm for constructing finite-state Muller automata that accept outer approximations to the exact controlled threshold-event language is presented, and it is shown that supervisors synthesized for the approximating automata achieve the control specifications when applied to the original hybrid system.

Dogruel [DOG 98] investigated the problem of partitioning the continuous state space into specific regions in order to simplify the analysis of the hybrid system. This paper shows that if the design is not carried out with care, some unpredictable behaviours may be encountered such as: different trajectory paths in a region, regional switching, uncertainty, and race situations.

Caines and Wei [CAI 98] present a definition of hierarchical hybrid control systems based on the notion of dynamic consistency. The notion of dynamic consistency is extended to hybrid systems to define the set of dynamically consistent hybrid partition machines. Between-block and in-block controllable hybrid partition machines are defined. The lattice of hybrid partition machines is also defined and investigated.

Nerode and Kohn [NER 93b] proposed a Multiple Agent Hybrid Control Architecture (MAHCA) as a software system for real-time implementation of distributed controllers. The architecture is based on principles of declarative control, concurrent programming and dynamical hybrid systems. The control action of each agent is determined by solving a relaxed optimization problem which is convex. Kohn and his coworkers have enhanced the proposed MAHCA for distributed intelligent control based on recent developments of hybrid systems theory and applications. They use novel ideas which include "chattering control", the "continualization" of the digital states to incorporate all the information about the system into a smooth manifold, and the use of infinitesimal differential geometry to formulate the control problem in the carrier manifold. See for example [KOH 94], [KOH 95], [KOH 96a,b,c], [GE 96] and references therein.

Lygeros and coworkers [LYG 96] present a methodology for designing hybrid controllers for large scale, multiagent systems based on optimal control and game theory. The hybrid design is seen as a game between two players: the control, which is to be chosen by the designer, and the disturbances that encode the actions of other agents, that is the actions of high level controllers or unmodeled environmental disturbances. The two players compete over cost functions that encode properties that the closed loop hybrid system needs to satisfy (e.g. safety). The control "wins" the game if it can keep the system safe for any allowable disturbance. The solution to the game theory problem provides the designer with continuous controllers as well as sets of safe states where the control "wins" the game. The sets of safe states are used to construct an interface to the discrete domain that guarantees the safe operation of the combined hybrid system. This approach has been used in air traffic management [TOM 98] and in control of automated highway systems [LYG 98].

Lemmon and Antsaklis [LEM 97a] provide insight into the integration of timed automata and robust control methods for the control of complex dynamical systems. Recent results in the computer science and robust control communities are presented and the integration of these methods are used for studying the stability and bounded amplitude performance of switched systems. Robust control methods have been used for hybrid control in [LEM 95], [BET 96], [LEM 97b].

Timed automata and hybrid automata have been used by several researchers for modeling and verification of hybrid systems. Alur and Dill [ALU 94] proposed timed automata to model the behavior or real-time systems over time. Timed automata are studied from the perspective of formal language theory (closure properties, decision problems, and subclasses). The theory is applied to automatic verification of real-time requirements of finite state machines. Alur et al. [ALU 95] introduced the framework of hybrid automata as a model and specification language for hybrid systems. Hybrid automata can be viewed as a generalization of timed automata, in which the behavior of variables is governed in each state by a set of differential equations. It is shown that the reachability problem is undecidable even for very restricted classes of hybrid automata. Two semidecision procedures are presented for verifying safety properties of piecewise-linear hybrid automata, in which all variables change at constant rates. A semidecision procedure for synthesizing controllers for hybrid systems modeled as linear hybrid automata has been presented in [WON 97]. The procedure has been implemented, and tested on the synthesis for various models of a steam boiler.

Puri and Varaiya [PUR 95] present two methods for verification of hybrid systems. The modelling formalism used is that of hybrid automata. Verification is based on abstracting the continuous dynamics in the hybrid system by simpler continuous dynamics. In the first method, a differential inclusion is replaced with a simpler differential inclusion; in the second method, the hybrid automaton is abstracted by a timed automaton. The methodology is illustrated by a train-gate-controller example. A variant of the differential inclusion method was used by Frick and Beard [FRI 98a] to address the automatic generation of discrete models for batch chemical unit operations including operating procedures and Relay Ladder Logic code. The combined model consists of discrete models for operation procedures and chemical unit operations and is amenable to Symbolic Model Checking. In [PUR 96] a method to compute an arbitrarily close approximation of the reach set of a Lipschitz differential inclusion is presented. Deshpande and Varaiya [DES 96] use nondeterministic finite automata to model the discrete behavior and differential inclusions to model the continuous behavior of hybrid systems. Viability refers to the ability of the system to perform an infinite number of discrete transitions and can be used to express safety and fairness properties over the system's state trajectories. To ensure viability, the system's evolution must be restricted so that the discrete transitions occur within specific subsets of their enabling conditions which are called viability kernel. Results pertaining to continuity properties of the viability kernel are given and conditions under which it can be computed in a finite number of steps are established. Finally, a hybrid controller that yields all viable trajectories is synthesized.

Henzinger and coworkers [HEN 98b] present a methodology for algorithmically analyzing nonlinear hybrid systems by first translating to linear hybrid automata, and then using automated model-checking tools. In a linear hybrid automaton, the analog environment is partitioned into a finite number of classes such that within each class, the analog variables are governed by a constant polyhedral differential inclusions. Two translation methods are presented. The first, called clock translation, replaces, when possible, constraints on nonlinear variables by constraints on variables with constant derivative equal to 1. This method is efficient but has limited applicability. The second method, linear phase-portrait approximation, conservatively overapproximates the phase-portrait of a nonlinear hybrid system using piecewise-constant polyhedral differential inclusions.

In [STU 98], a framework for modelling and analysis of discretely controlled continuous processes with at least piecewise continuous dynamics is presented. The framework is interfaced to HYTECH tool [HEN 95], thus providing formal verification facilities of logic controllers, including the use of timers. It offers hierarchical structure to model a process based on Timed Condition/Event Systems [ENG 95] on the lowest level, and it includes facilities to edit control programs compliant to the IEC1131-3 standard and to generate discrete approximation of continuous process models. Furthermore, it includes an integration-based discretization procedure to approximate continuous dynamics by timed discrete models. The framework is illustrated by means of a controlled chemical batch-reactor.

Numerous simulation tools have been proposed for the simulation, verification and implementation of hybrid systems. SHIFT proposed by Deshpande et al. [DES 98] is a programming language for describing dynamic networks of hybrid automata. The SHIFT models offers the proper level of abstraction for describing complex applications such as automated highway systems whose operation cannot be captured easily by conventional systems. Henzinger and Ho [HEN 95] proposed HYTECH as an automatic tool for analyzing hybrid systems. Daws et al. [DAW 96] developed Kronos as a verification platform for complex real-time systems. Taylor and Kebede [TAY 96] developed MATLAB tools for modelling and simulation of hybrid systems. Van Beek and Rooda [BEE 98] highlighted the facilities for discontinuity specification in the combined continuous-time/discrete-event language.

Fritz et al. [FRI 98b] present an object-oriented open framework that is adapted to the simulation of recipe driven production. This framework integrates different simulation strategies (purely discrete, timed discrete, continuous-discrete) and enables consistent modelling regardless of the simulation strategy used. The simulation model is divided into a generic and a simulator-specific part. The generic simulator-independent part consists mainly of structural information, including the structure of the recipe and the interconnections of plant units. In contrast, the specific model part contains model descriptions following a certain modelling paradigm., e.g. differential equations or finite automata.

Hohmann and Zanne [HOH 98] proposed a comparison of eight hybrid-simulation software packages. The comparison criteria are: type of simulation tool (general or special purpose), type of modeling language used (programming language, simulation language, block oriented), existence of graphical interface, target execution language, modularity, hybrid simulation method used (whether continuous and discrete parts are strictly distinguishable or not), event detection and treatment when crossing the transition set, type of continuous differential equation solver and implementation of Petri nets.

3. Petri nets in Hybrid Control Systems

Petri nets have been used extensively as a tool for modeling, analysis and synthesis for discrete event systems. For DES control, Petri nets modeling formalism offers some advantages over finite automata, and it is also useful for hybrid systems control. In the following, we review a number of approaches involving extensions of Petri nets to hybrid systems. The approaches outlined are arranged from papers describing continuous and hybrid Petri nets, to papers combining Petri nets with differential equations, and then papers implying extensions of timed Petri nets or Condition/Event nets. Finally, some hybrid applications based on the use of Petri nets are outlined.

A major step in the effort to enhance the modeling power of Petri nets has been their extensions known as Continuous Petri Nets [DAV 87]. The basic idea is to consider that the marking of places can be a real number instead of an integer. Continuous Petri nets are thus approximations to discrete-event systems allowing, basically, faster simulation of the latter without sacrificing accuracy. Various timed continuous PN models have been defined and they correspond to different calculations of the firing speeds associated to the transitions [DAV 97]. A method for choosing the parameters of a continuous Petri net based on the use of a hybrid automaton is proposed in [ELF 98].

Hybrid Petri Nets proposed by Le Bail et al. [LEB 91] is a combination of ordinary and continuous Petri nets. This model can treat integer variables together with real variables and symbolic variables usually encountered in other models of hybrid systems. It inherits all the modeling facilities of Petri nets such as the ability to capture concurrency, synchronisation and conflicts, allowing to model systems with continuous flows and linear evolutions in an intuitive way. Hybrid Petri Nets provide some basic analysis tools that are limited to the study of some structural properties of the autonomous model such as invariants.

Allam and Alla [ALL 98] present a procedure for constructing the hybrid automaton associated with a hybrid Petri net, in order to benefit from the modeling power of the latter and the analysis power of the former. This procedure, which is implemented in HYTECH [HEN 95], uses forward analysis to compute the reachable state of hybrid automata, and to characterize the periodical functioning of the associated Hybrid Petri net. Different performance measurements can be computed for the identified periodical functioning modes such as: mean marking of a place, mean firing frequency of a transition, and the mean dwelling time of tokens in a given place. Another approach to investigate the periodicity of Hybrid Petri Nets is proposed by [KOM 98] to evaluate the period of hybrid closed manufacturing lines. The hybrid state space model is represented using conventional algebra for the continuous subsystem and the minplus algebra for the discrete subsystem. Pettersson and Lennartson [PET 95] used Bond graphs to verify systems described by hybrid Petri nets and compared hybrid Petri net and switched Bond graph modeling using a process example.

Many extensions have been proposed to further improve the modeling power of hybrid Petri nets. Batch Petri nets [DEM 93], for example, are well adapted for modeling, simulation and performance evaluation of single-product high-throughput production lines and accumulation systems. This model is based on the definition of a set of parts on a conveyor, named internal coherent batch, and allows to formalize the circulation of accumulated and non accumulated products on a conveyor. A colored version of batch Petri nets is proposed in [CAR 98a] to apply in the case of flexible, multi-product hybrid and high throughout production lines. In this paper, a generic transportation model, called virtual colored conveyor, is given for illustration.

Hybrid Flow Nets, an extension of the hybrid Petri nets to nonlinear systems, have been proposed by Flaus [FLA 97]. This model is well suited for modeling transformation processes such as chemical or food processes with hybrid characteristics, namely the ones operating in batch mode. A supervisor synthesis approach for hybrid flow nets has also been developed [FLA 98]. This approach builds on the Petri net supervisory synthesis method proposed by Yamalidou and coworkers [YAM 96], and based on the notion of place invariants

The discrete part of hybrid nets has been extended by Giua to obtain high-level hybrid Petri nets [GIU 96]. This new model, which merges the concepts of high-level Petri nets [MUR 89] with continuous nets, is characterized by the use of structured individual tokens in the discrete part of the net. High-level hybrid Petri nets can represent jumps in the state space and switches in the dynamics, both autonomous and controlled. Classical Petri net concepts, such as the firing vector and the incidence matrix can be generalized to this model and used to derive the evolution equation [GIU 98].

Demongodin and Koussoulas [DEM 98a, b] considered a new extension of continuous Petri nets, called Differential Petri Nets. Through the introduction of the differential place, the differential transition, and suitable evolution rules, it is possible to model concurrently discrete-event processes and continuous-time dynamic processes, represented by n linear first order differential equations. The marking of the differential place is positive, negative or null real representing a state variable of the continuous system that is modeled.

Drath [DRA 98] presented a class of Petri nets which attempts to combine the advantages of hybrid Petri nets and object-oriented paradigms to manage the complexity of hybrid systems.

High Level Petri Nets including a set of differential equations were proposed by Vibert et al. [VIB 97] to model batch processes taking into account fluctuations of continuous variables. A model combining Predicate/Transition Petri nets and differential equations has also been developed by Champagnat et al. [CHA 97].

Peleties and DeCarlo [PEL 94] presented a model based on the work by Ramadge [RAM 90] on the periodicity of symbolic observations of piecewise smooth discrete-time systems. The hybrid model is suitable for Petri net based symbolic analysis of hybrid systems; the continuous plant is approximated by a Petri net and a supervisor consisting of two communicating Petri nets controls the behavior of the open plant.

Lunze and coworkers [LUN 97] proposed a model where Petri nets are used as a discrete event representation of the continuous variable system; the system and the interface are represented by a Petri net and the supervisor represents a mapping of the output event sequence into the input event sequence.

He and Lemmon [HE 98] proposed an extension of Alur's hybrid automaton [ALU 93] in which a timed Petri net is used to model a hybrid system. The resulting modeling framework, called a programmable timed Petri net (PTPN), is used to model hybrid systems obtained by switching between collections of linear time-invariant plants. Each subsystem of the switched system is represented by means of a local timer. Unfoldings of PTPN is used to identify equivalence classes of configurations from which fundamental cycles in the PTPN's reachability graph can be identified. These cycles are then used to form two different types of linear matrix inequalities whose feasibility ensure either the Lyapunov stability or the switched system's uniform ultimate bounded behavior.

In [KOU 98a] programmable timed Petri nets are used to model and control hybrid systems. In particular, the stability and supervisory control of hybrid systems are addressed and efficient algorithms are introduced. Sufficient conditions for the uniform ultimate boundness of hybrid systems composed of multiple linear time invariant plants which are switched between using a logical rule described by a Petri net are presented. For supervisor control design (see also [KOU 98b]), the transfer of the continuous state to a region of the state space under safety specifications on the discrete and continuous dynamics is addressed. The switching policy is embedded in the dynamics of the underlying Petri net structure and the supervisors are described by Petri nets. The discrete specifications are expressed in terms of linear constraints on the marking vector and are satisfied by applying supervisory control of Petri nets based on place invariants [MOD 96]. The hybrid system switches from a subsystem to another, in a way that the state gradually progresses from one equilibrium to another towards the desired target equilibrium. The supervisory control algorithm is designed to allow switchings to occur only on the intersection of the invariant manifolds. Finally, in the case when the continuous dynamics are described by first order integrators, the design algorithm is formulated as a linear programming problem.

Chen and Hanisch [CHE 98] extended net/condition event systems [RAU 95] to hybrid net condition/event systems so that they can be used to model batch processes in a modular way. A semi-decidable algorithm for analysis of the state reachability for the hybrid net condition/event system is proposed based on constructing the discrete part of the net. During this construction, a pruning criterion and operation is applied to avoid enumerating all the state of the system.

Cook and Evans [COO 98] present an approach to the description of hybrid systems based on Petri nets with C/E net semantics. They sketch the application of a verification technique based on the computation of fixed-points of predicate transformers over system states in a way to provide a basis for symbolic model checking. The work draws principally on research into interval timed nets [AAL 93] and automata-based models of hybrid systems [ALU 95]. With the proposed technique, the reachability problem for hybrid systems cannot be solved in general.

The Grafcet model, drawing inspiration from Petri nets, is an international standard used for the specification of logic controllers in manufacturing systems [DAV 95], [ZAY 97]. This model is the basis of the Sequential Function Charts (SFC) international standard used for the implementation of logic controllers. Requirements for extending Grafcet to allow the specification of hybrid controllers are discussed in [GUI 98]. A precise definition of the behavior of Grafcet that establishes the basis for multi-model temporal integration with other continuous based models in the frame of hybrid control systems is given in [LES 98]. An approach is also proposed to verify hybrid systems whose discrete controllers are specified using Grafcet [ZAY 96]. In this approach, the plant continuous dynamics is approximated by means of timed transition models. High-Level Grafchart [JOH 98b] is an extension of Grafcet that uses High-Level Petri net based programming constructs (parameterization, methods and message passing, object tokens, and multi-dimensional charts) to represent recipes and resource allocation for multi-purpose batch plants.

Many hybrid applications are illustrated using Petri nets. Hohmann and Zanne [HOH 98] use interpreted Petri nets associated with particular operators to model an induction machine and its associated gate turn-off converter. Sechilariu and Ratoi-Cozma [SEC 98] show the feasibility of a strategy for controlling a static converter by microcontroller using Petri nets. A simple control system of a room heating by a thermostat was used by Gernich and Schuart, [GER 98] to illustrate an approach for modelling, simulation and verification of hybrid systems using hierarchical colored Petri nets.

In [CHA 98], a benchmark example corresponding to a gas storage unit in which the continuous view involves non-linear and differential equations is given. The discrete system, which describes the various configurations of equipment, is rather simple. The objective of this benchmark is to compare various approaches for modelling hybrid systems in order to derive some useful guidelines for academics and engineers. The control law of the storage unit is expressed by a set of constraints, and the production policy and customer demand are defined by constants that change their values at discrete time points. Three modeling and simulation approaches are compared on the basis of this example: hybrid flow nets [FLA 97], integration of a Predicate/Transition Petri Nets with sets of differential and algebraic equations [CHA 97], Petri Nets and differential algebraic equations [VIB 97].

The goal of the work presented by Decknatel and Schnieder [DEC 98] is to propose a new approach for the integrated description and study of railway systems using hybrid Petri nets. Through a survey of the state of the art in railway modeling and of the requirements imposed by the hybrid nature of this application domain, the authors suggest that the introduction of hybrid nets can close some of the gaps and lead to improvements in terms of design, simulation, and verification of railway systems.

4. Conclusion

This paper summarized some of the research results of the recent significant research activity in the area of hybrid control systems. Control systems that contain both continuous and discrete dynamics have been studied off and on for the past forty years and related results are the results on bang-bang control and on sliding mode control among others. This recent research activity in hybrid control systems follows closely and has been motivated in part by the development of research results in the control of discrete event systems that occurred in the 80's and of adaptive control in the 70's and 80's and of the renewed interest in optimal control formulations in sampled-data systems and digital control. At the same time there has been growing interest in hybrid systems among computer scientists and logicians with emphasis on verification of designs that involve hybrid automata. In this paper the emphasis was placed on hybrid control with special attention paid to approaches that involve Petri nets. It should be noted that due to paper length limitations the list and descriptions of approaches are certainly not complete. This paper therefore represents a restricted view of the hybrid control systems field taken from a window in time. It is hoped however that it will provide some useful insight of the main approaches to hybrid control systems and it will be a useful resource to anyone interested in hybrid systems modeling, analysis and design.

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